AN ATTEMPT TO MODEL THE GLOTTIS AS A VAN DER POL OSCILLATOR

Some say it's a stone, other say it's a bird...
Indeed it is an egg!!

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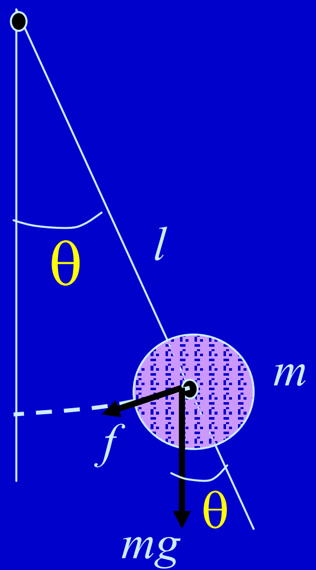
PRINCIPLES OF HARMONIC OSCILLATOR

Gravity Pendulum



$$\ddot{\theta}+\omega_0^2=0$$

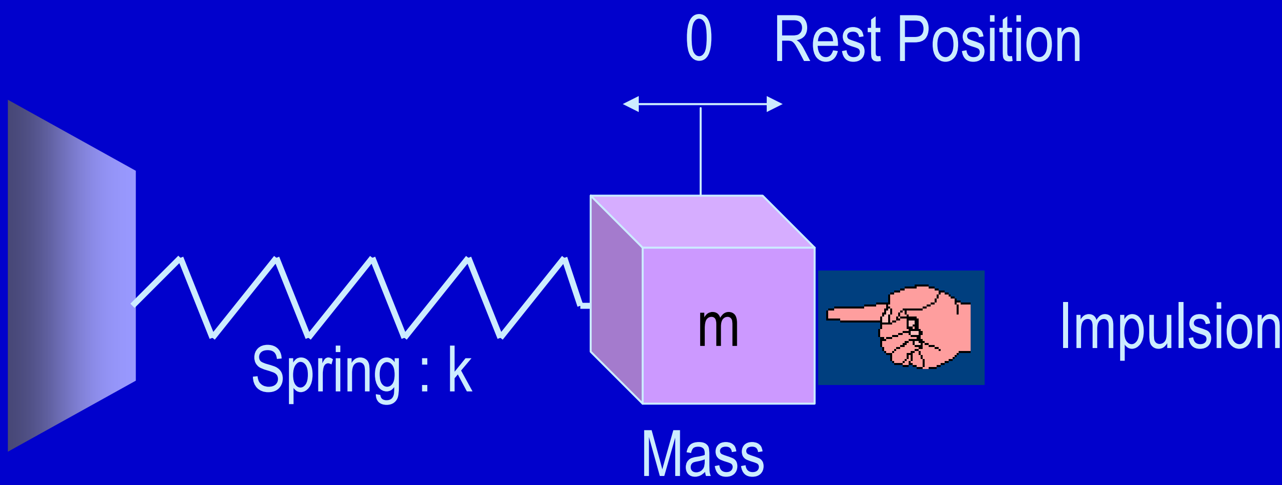
$$\omega_0^2=\frac{g}{l}$$

All physical systems characterized by a parameter u(t) satisfying the differential equation :

$$\ddot{u}+\omega_0^2 u=0$$

are called harmonic oscillators

SPRING-MASS OSCILLATOR

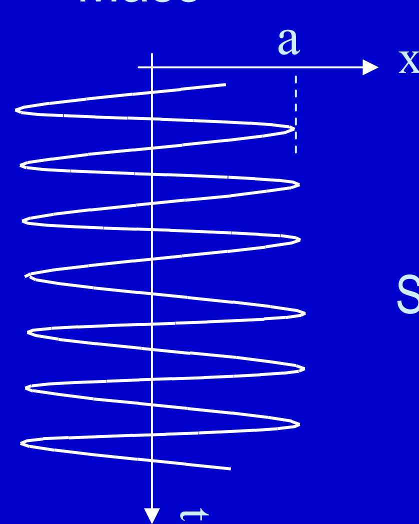


$$m\ddot{x}+kx=0$$

$$\omega_0^2=\frac{k}{m} \Rightarrow \ddot{x}+\omega_0^2x=0$$

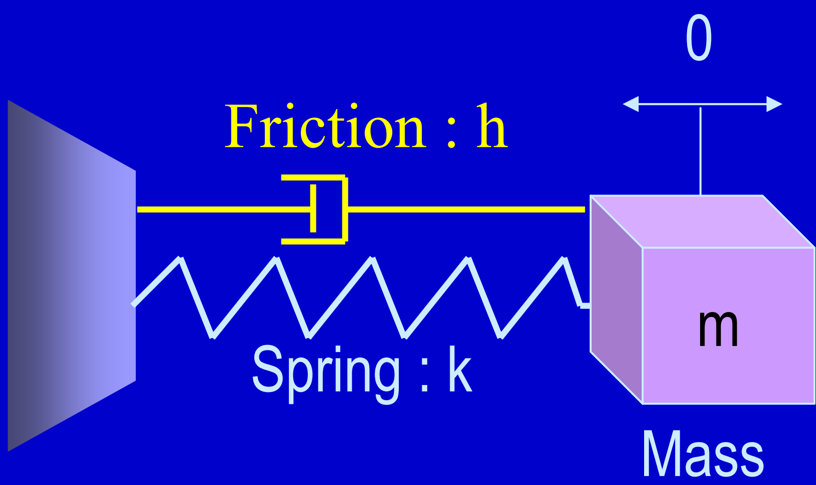
$$x=a\cos(\omega_0 t+\varphi)$$

$$F_0=\frac{\omega_0}{2\pi}$$

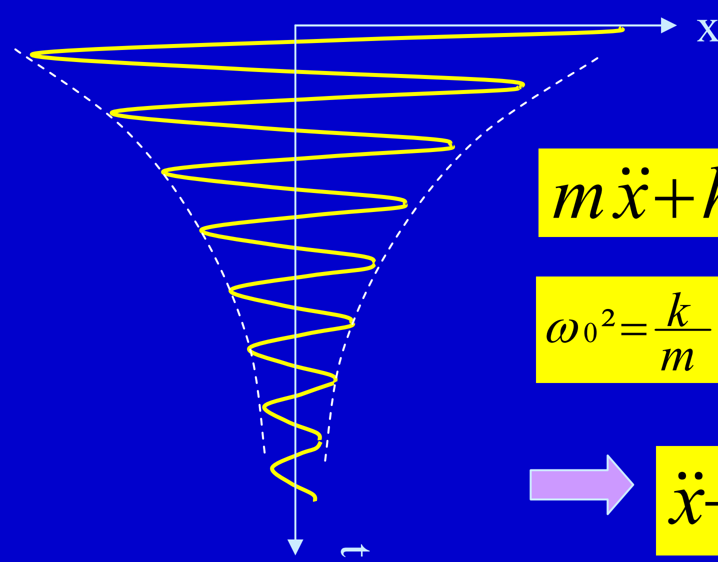


Sinusoidal Oscillation

SPRING-MASS-FRICTION OSCILLATING CIRCUIT



Damped



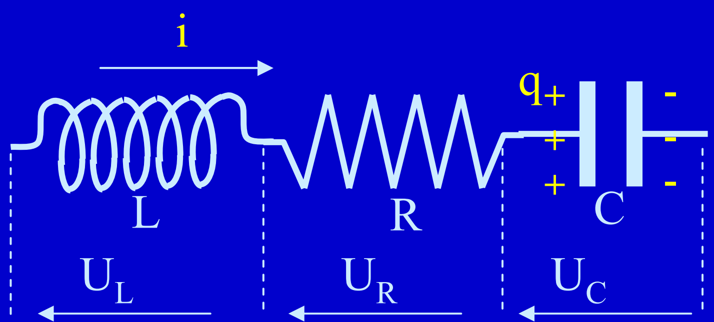
$$m\ddot{x}+h\dot{x}+kx=0$$

$$\omega_0^2=\frac{k}{m}$$

$$\gamma=\frac{h}{m}$$

$$\ddot{x}+\gamma\dot{x}+\omega_0^2x=0$$

ELECTRICAL ANALOGY



$$L\,\ddot{q} + R\,\dot{q} + \frac{q}{C} = 0$$

$$\omega_0{}^2 = \frac{1}{LC} \qquad \gamma = \frac{R}{L}$$

Mechanic $\boxed{\ddot{x} + \gamma\dot{x} + \omega_0{}^2 x = 0}$

$\ddot{q} + \gamma\dot{q} + \omega_0{}^2 q = 0$ Electric

Variables $\boxed{x \leftrightarrow q}$ $\boxed{v = x' \longleftrightarrow i = q'}$ $\boxed{f \leftrightarrow u}$

Parameters $\boxed{m \leftrightarrow L}$ $\boxed{h \leftrightarrow R}$ $\boxed{k \longleftrightarrow 1/C}$

HARMONICS OSCILLATORS

In a conservative system :

The total energy is constant

The x strength is

$$x = a\cos(\omega_0 t + \varphi)$$

ENERGY

potential ⟷ kinetic ⟷ potential

$$\frac{1}{2}\frac{x^2}{k} \qquad \frac{1}{2}mv^2 \qquad \frac{1}{2}\frac{x^2}{k}$$

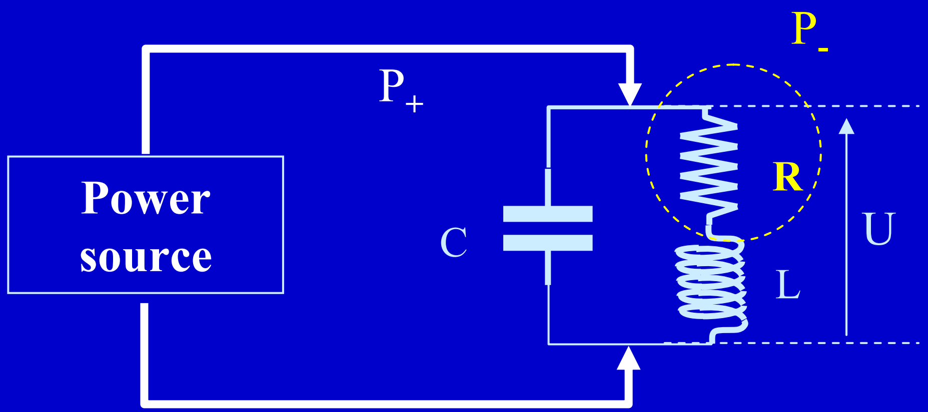
mass

spring

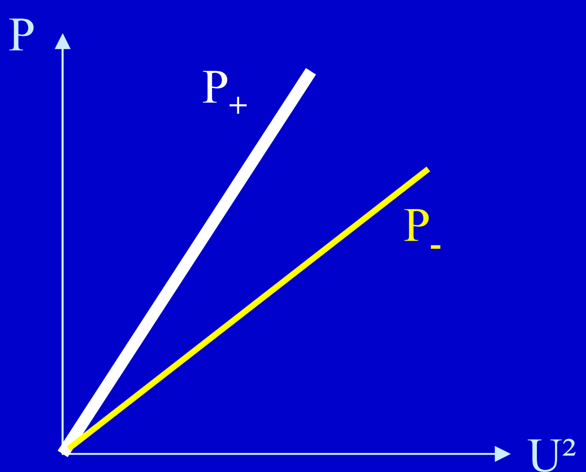
The frequency is

$$F = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

SELF-SUSTAINED OSCILLATION : CONDITIONS

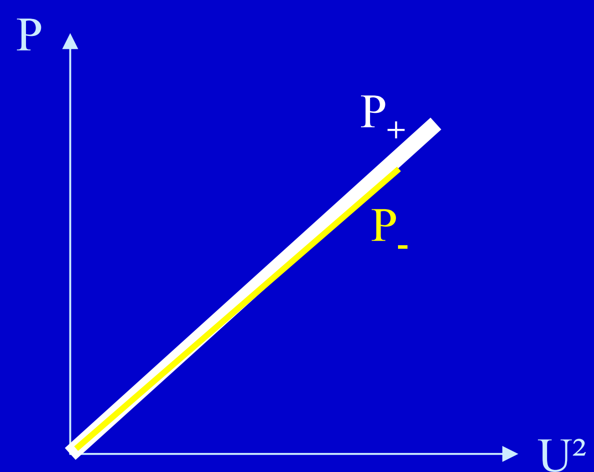


$$P_{-} = \frac{1}{2}\frac{U^2}{R}$$



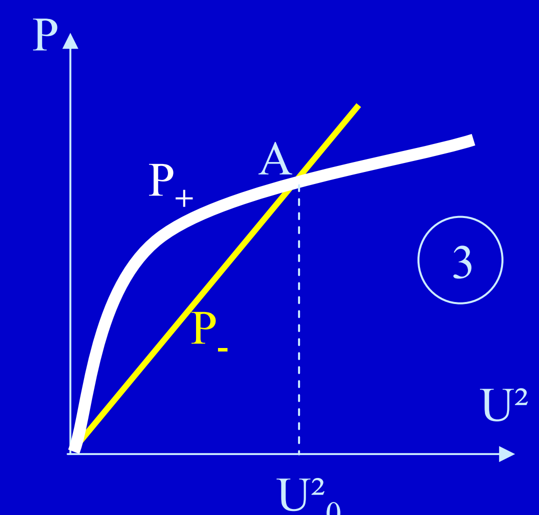
$P_{+} > P_{-}$: Amplitude increase indefinitely

1



$P_{+} = P_{-}$: Amplitude indetermined

2



Dynamic stability ; Self-sustained oscillation with stable amplitude U_0

SELF-SUSTAINED OSCILLATION
Van Der Pol's equation

The coefficient of friction γ is a function of the amplitude x of the oscillation. The parameter γ is negative for the small amplitudes and positive for the large amplitudes. Only the module, and not the sign of the amplitude x has an importance. Therefore $\gamma(x)$ is in x^2 .

$$\boxed{\ddot{x}+\gamma\dot{x}+\omega_0^2 x=0}$$

$$\gamma(x)=-\gamma_0\left(1-\frac{x^2}{x_0^2}\right)$$

$\gamma_0 > 0$; x_0 reference amplitude

Then :

$\gamma < 0$ for $x^2 < x^2_0$

$\gamma > 0$ for $x^2 > x^2_0$

$$\boxed{\ddot{x}-\gamma_0\left(1-\frac{x^2}{x_0^2}\right)\dot{x}+\omega^2 x=0}$$

Unity of amplitude $x_0\sqrt{\dfrac{\omega}{\gamma_0}}$

Unity of time $\dfrac{1}{\omega}$

$$\Rightarrow \quad \boxed{\ddot{x}-(\varepsilon-x^2)\dot{x}+x=0}$$

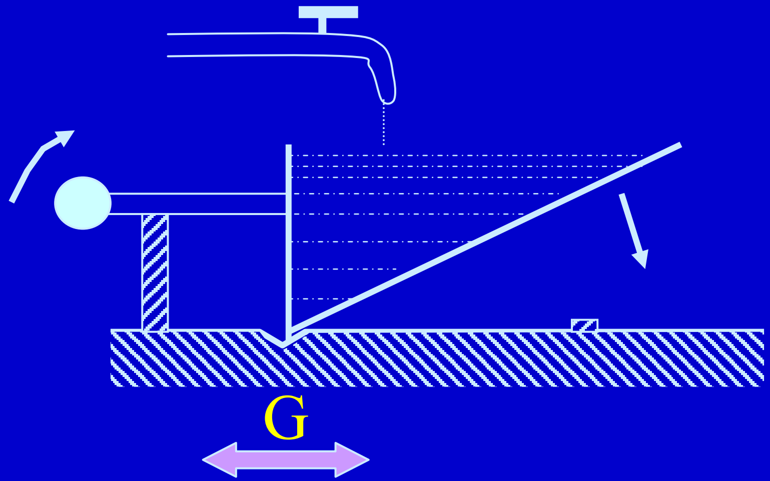
$$\varepsilon=\frac{\gamma_0}{\omega}$$

Problem

« Experimentally, phonation tends to « kick in » and « kick out » in a more abrupt way than small amplitude theory would predict. Furthermore, the kicking in may occur at a higher value of pressure than the kicking out, as reported by Baer 1975, suggesting a hysteresis (memory) for oscillation having previously been on or off. »

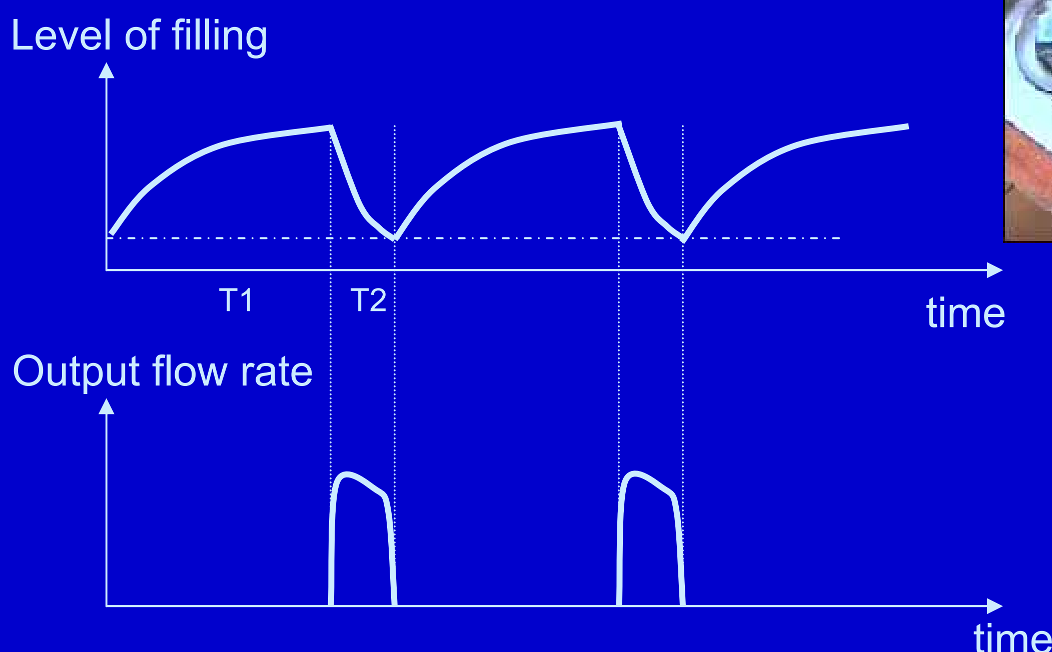
Ingo R. Titze.*Phonation threshold pressure: A missing link in glottal aerodynamics JASA 1992;91:2926-2935.*

RELAXATION OSCILLATOR



Seesaw.

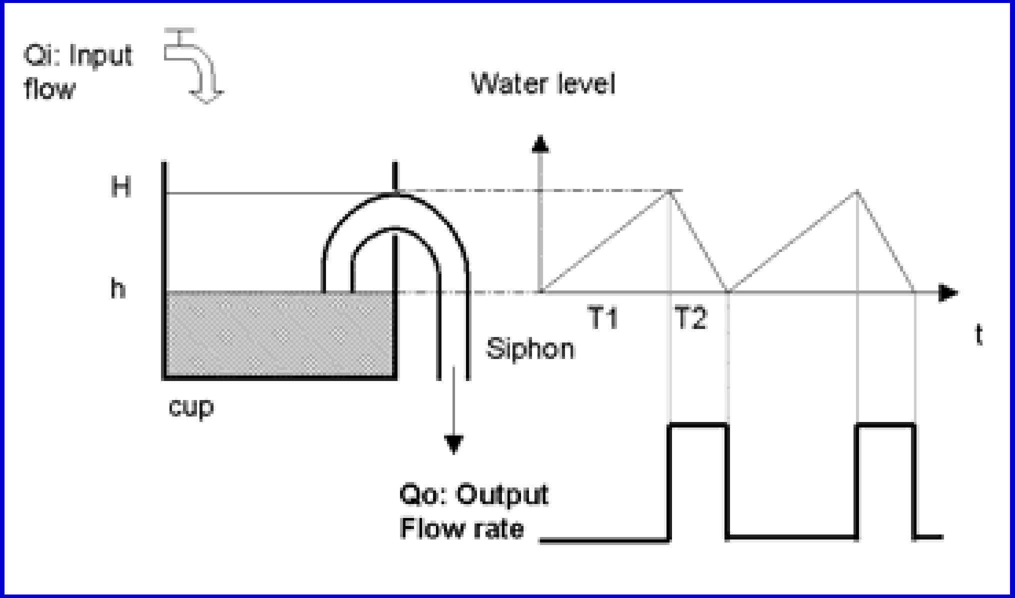
Swing when the G center of gravity passes by the plan containing the axis of rotation

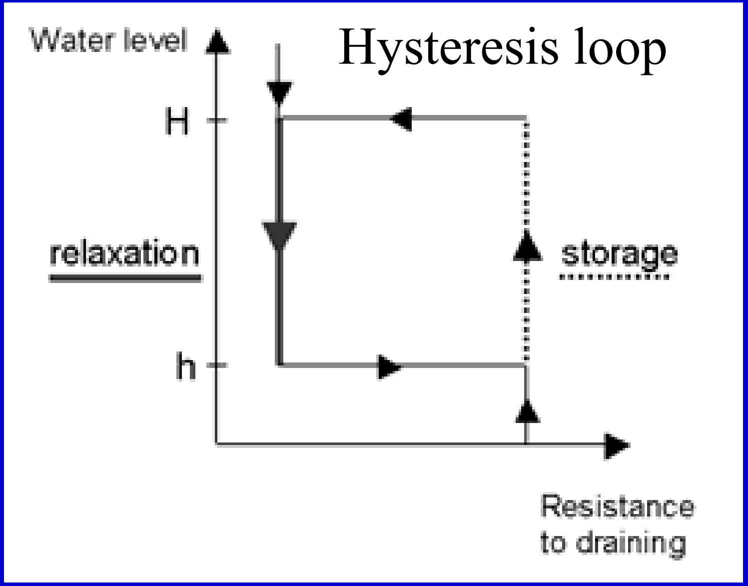


The frequency is in direct relation with the flow of filling and draining

ANOTHER TYPE OF RELAXATION OSCILLATOR

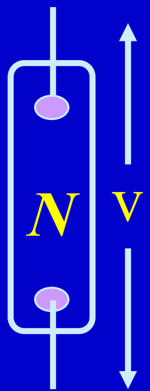
Tantalus cup used for time measurement during the Roman Empire





When water reaches the level H, the siphon primes, the tank is quickly emptied with a flow higher than that of the filling, down to the level h of draining

NONLINEAR ELECTRICAL RESISTANCE



The non-linear N organ basically presents two resistances R1 and R2 without transition according to the characteristics fig. 1 and the cycle fig.2

$$0 \xrightarrow{R2} V2 \xrightarrow{R1} V1$$

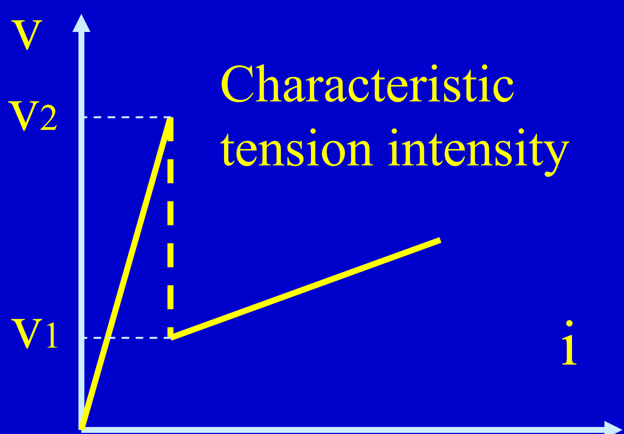
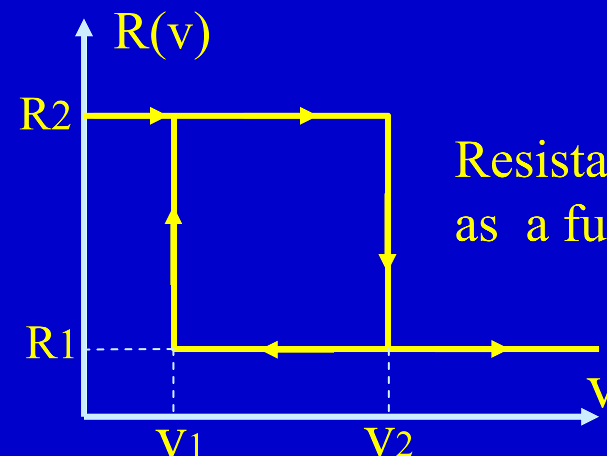


Fig. 1



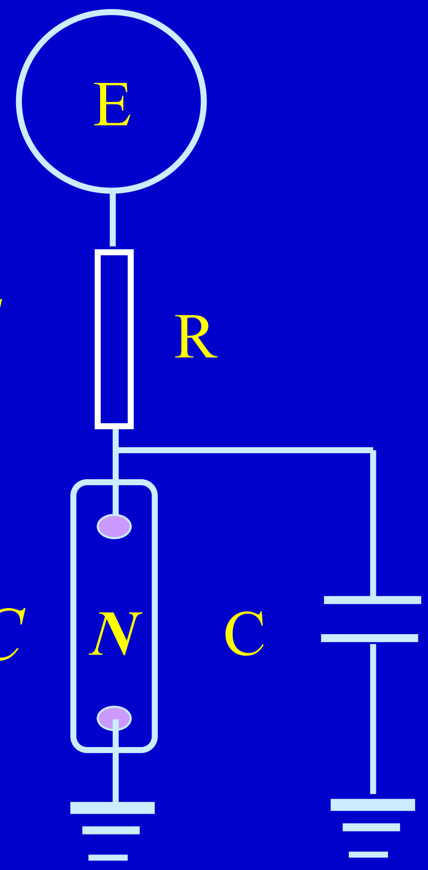
Resistance cycle as a function of v

Fig. 2

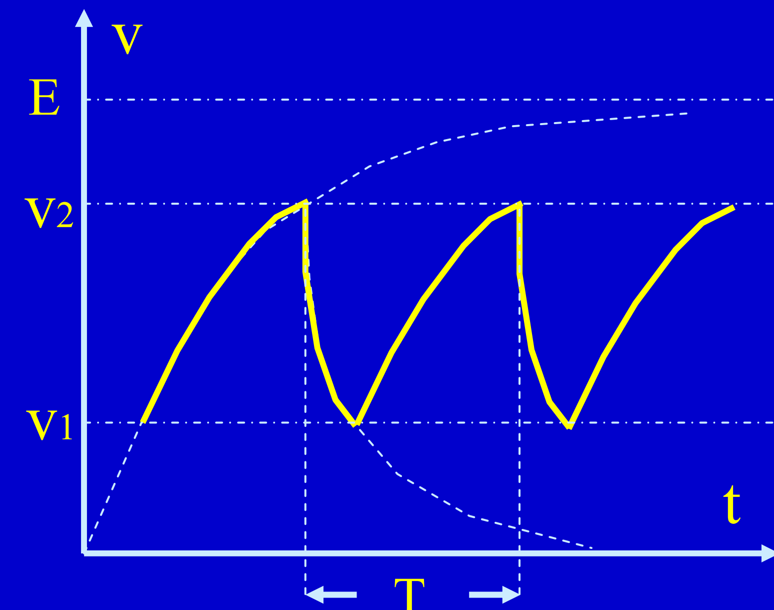
SIMPLEST ELECTRICAL RELAXATOR

$\theta_1 \approx RC$

$\theta_2 \approx R_1 C$



Neon Oscillator



$$\boxed{T = \theta_1 \log \frac{E - V_1}{E - V_2} + \theta_2 \log \frac{V_2}{V_1}}$$

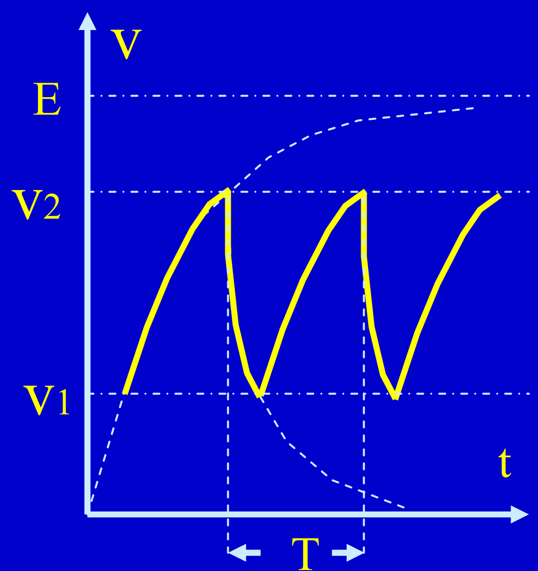
OSCILLATION INTERVAL

Relaxation oscillation require two conditions

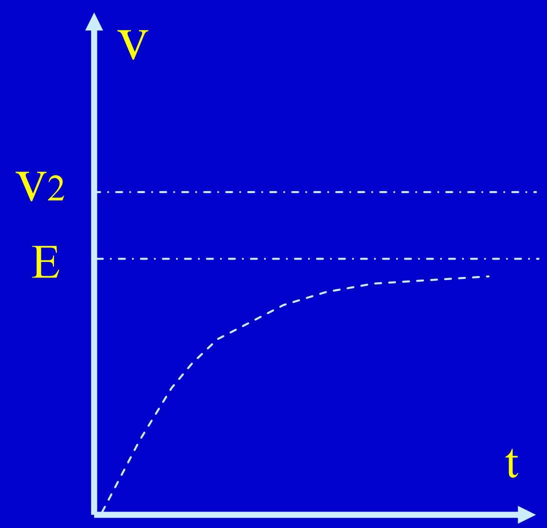
1) The system must progress and reach the high threshold -« Onset »- (V2)

2) The relaxation must reach the low threshold -« Offset »- (V1)

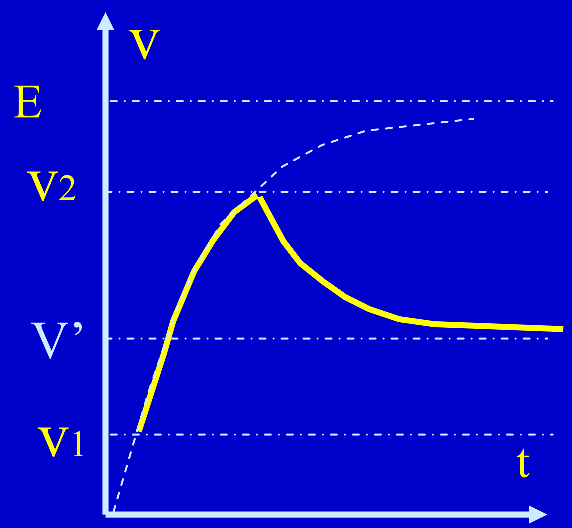
as in the phonation conditions (Onset and offset of phonation)



$$\frac{ER2}{R+R2} > V2 \qquad \frac{ER1}{R+R1} < V1$$

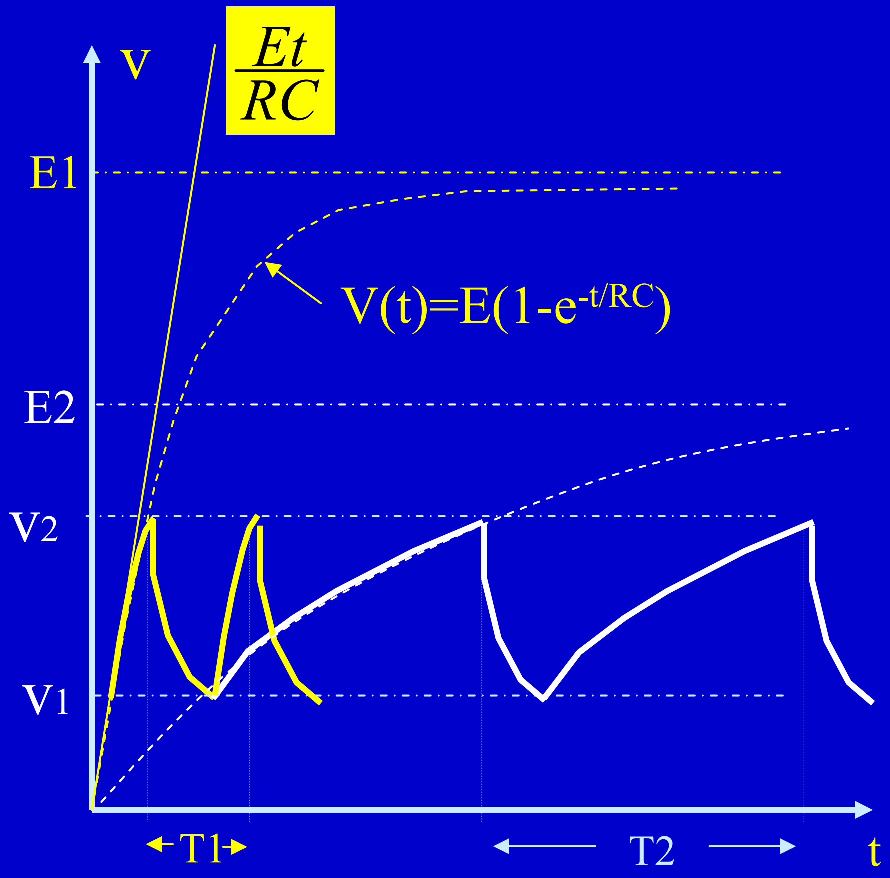


$$\frac{ER2}{R+R2} < V2$$



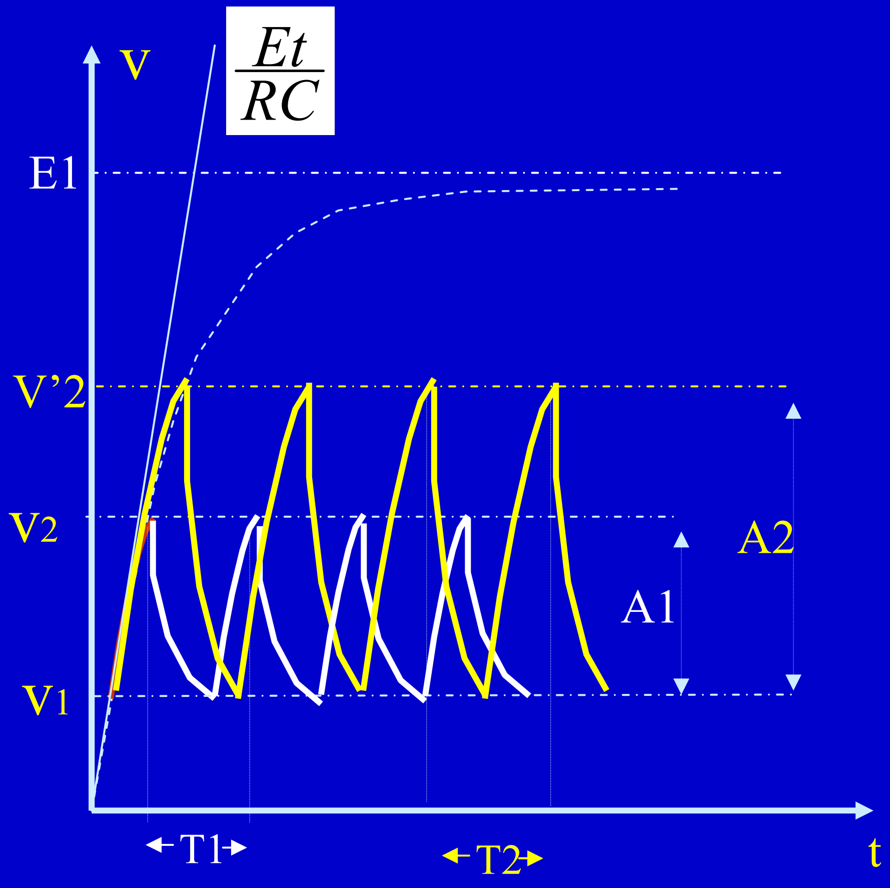
$$V' = \frac{ER1}{R+R1} > V1$$

FREQUENCY CONTROL



The frequency increases with E and 1/C

AMPLITUDE CONTROL



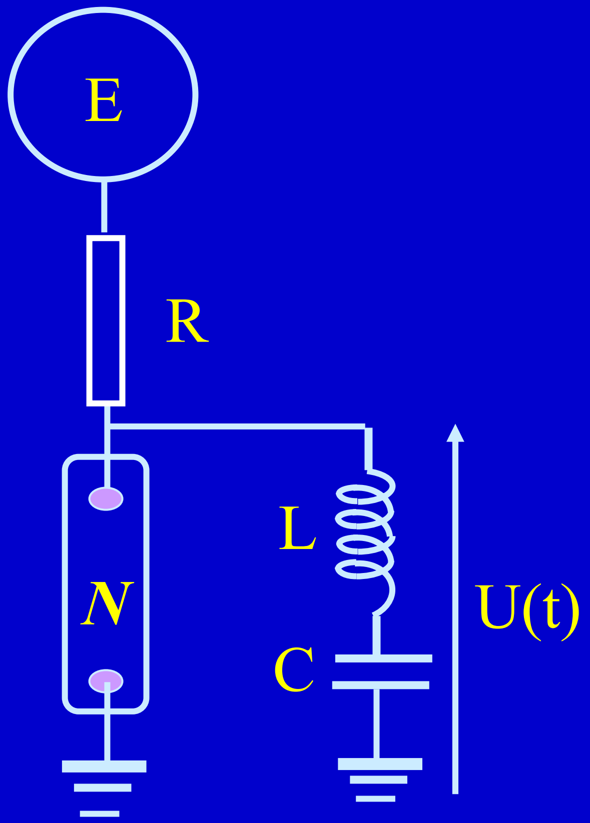
In the « neon » modelling, when V2 increases to V'2, then A2>A1. One must notice that the period increases T2>T1.

In laryngeal vibration, the increase of the adduction force has two consequences

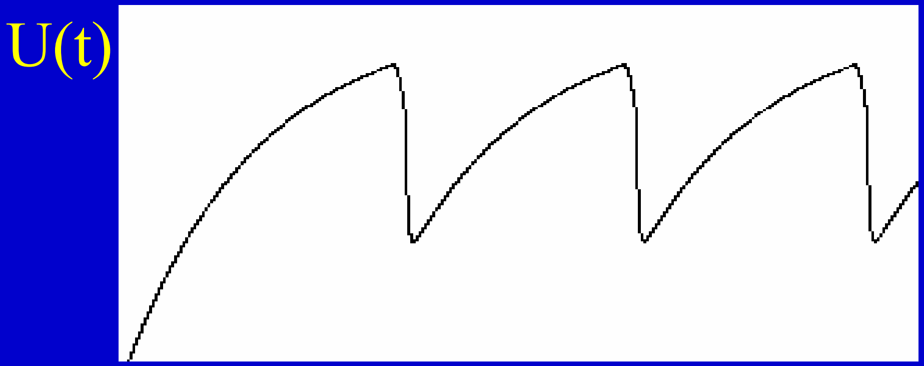
1) Increase of the opening threshold, thus increase Amplitude
2) Increase of the stiffness of the spring (eq. 1/C), thus increase of the frequency

NON LINEAR RESISTANCE RELAXATOR AND OSCILLATING CIRCUIT

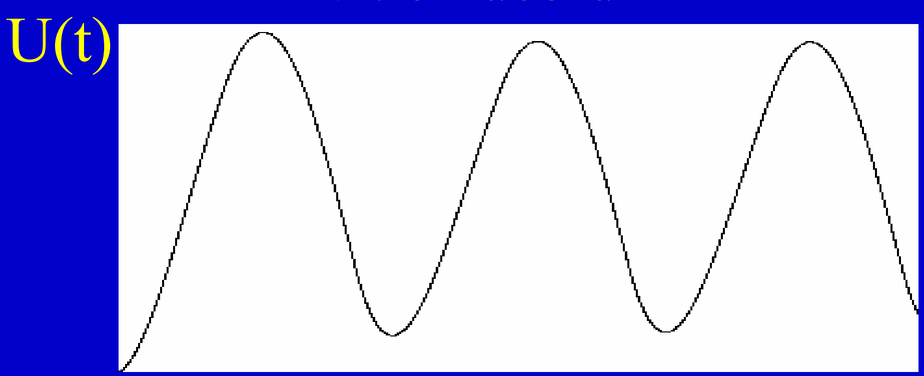
Numerical simulation



L = 0 : relaxator



L > 0 : sinusoïd



Simulation by hysteresis method

VAN DER POL EQUATION

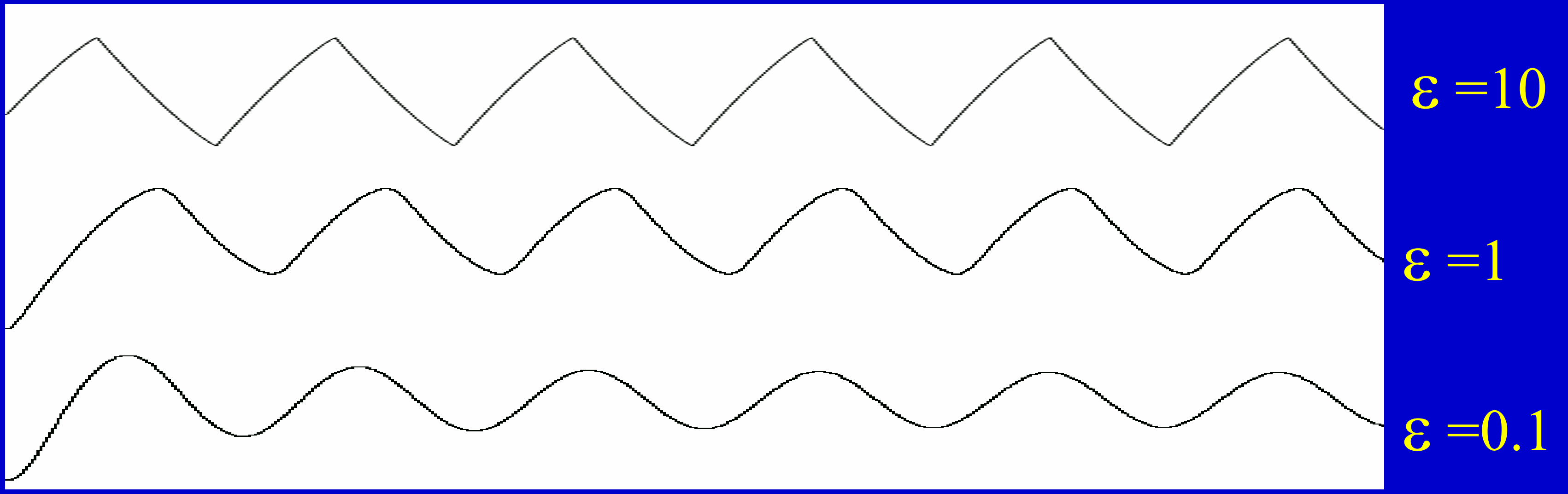
Non linear resistance $R(i) = -\rho(1 - \beta i^2)$

$y = i\sqrt{\beta}$

$x = \frac{t}{\sqrt{LC}}$

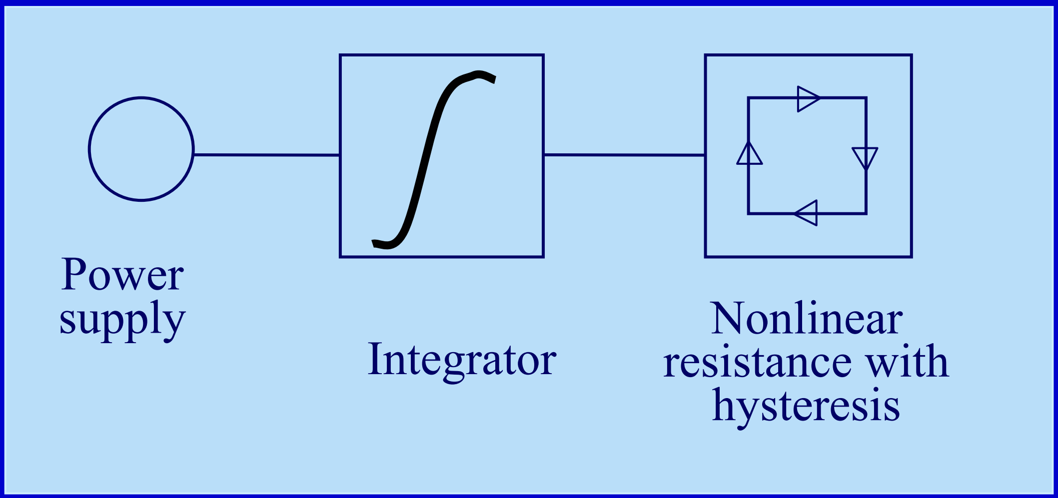
$\varepsilon = \rho\sqrt{\frac{C}{L}}$

$$y'' - \varepsilon(1 - y^2)y' + y = 0$$

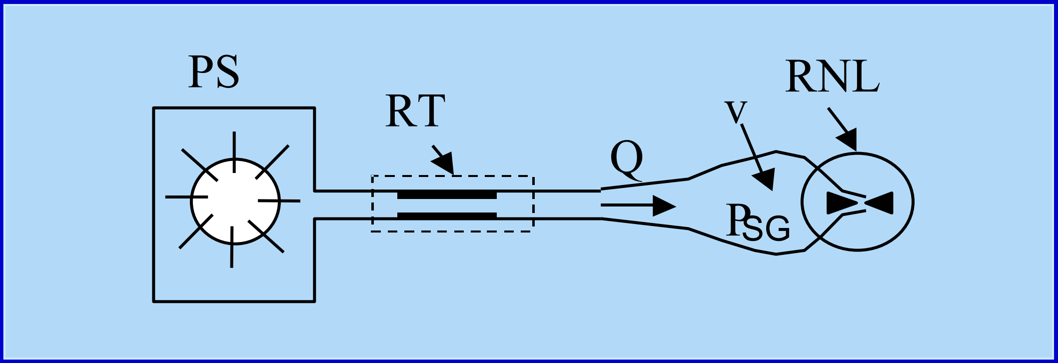


Simulation by Van Der Pol method

The principles of relaxation oscillator



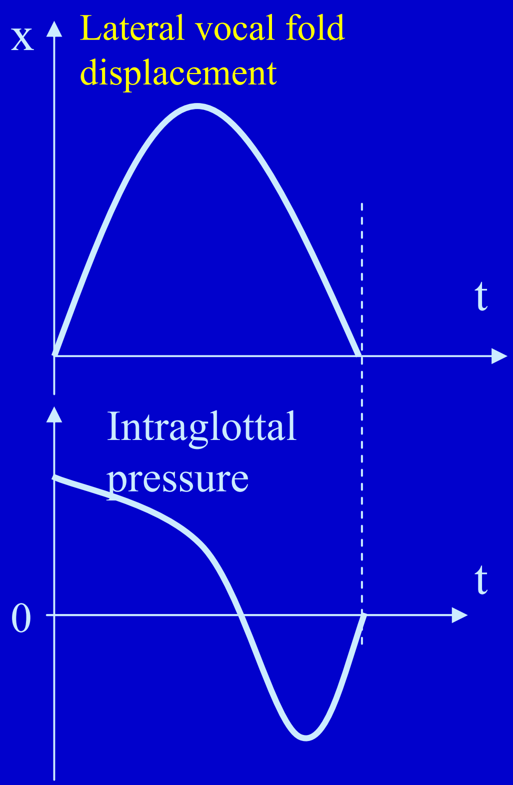
In a general way, the phenomenon of relaxation oscillation is demonstrated by any system presenting three characteristics : power supply, integrator and nonlinear resistance with hysteresis behavior

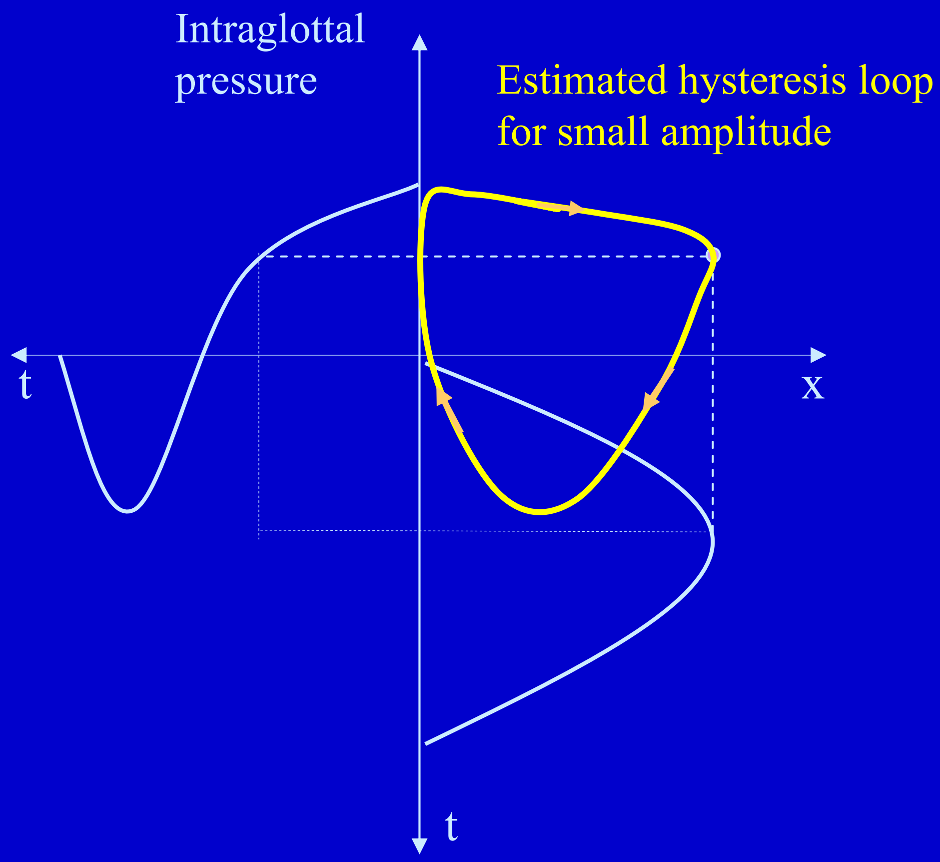


Regarding the phonation system, the intraglottic space can be grossly approximated by a box able to integrate airflow.

The glottis may present a nonlinear resistance with hysteresis to the airflow

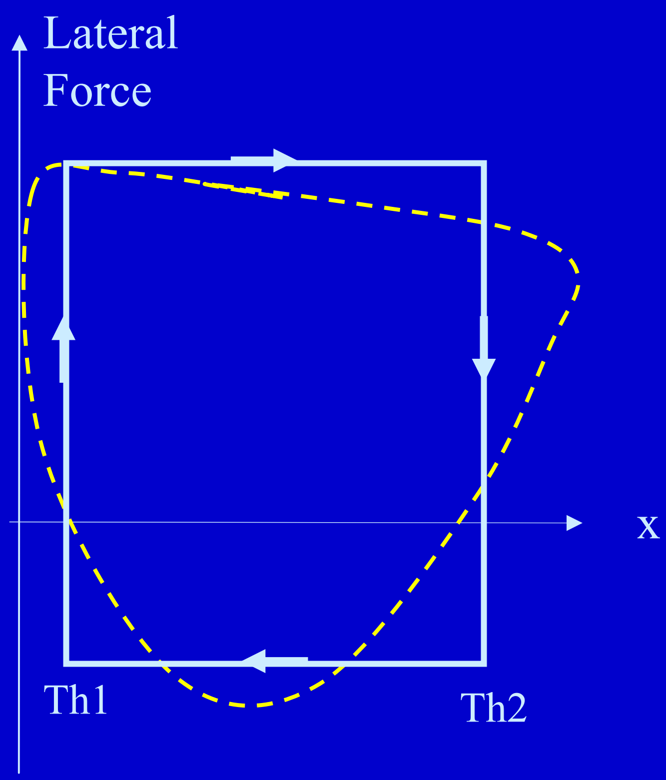
The glottis as a nonlinear resistance with hysteresis ?

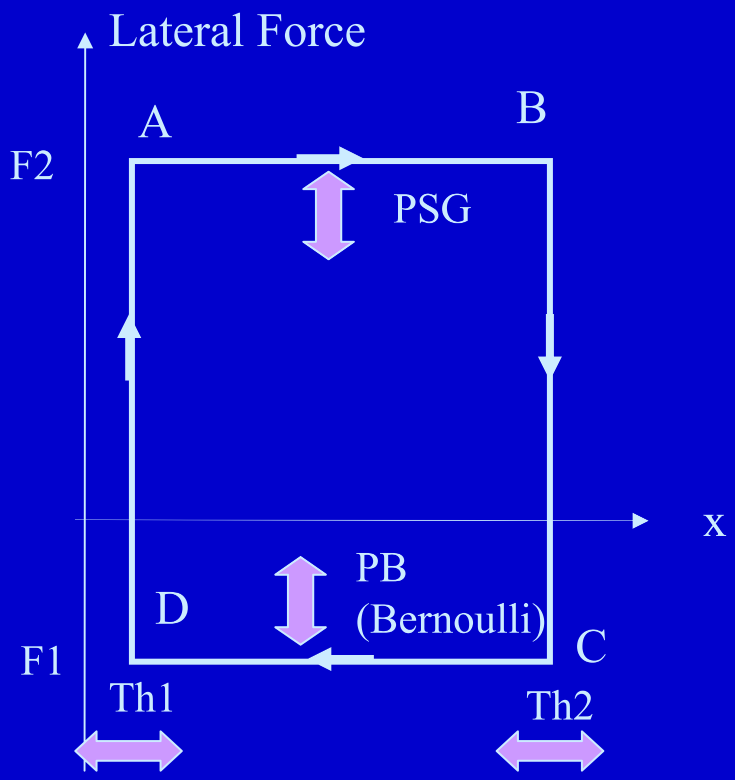




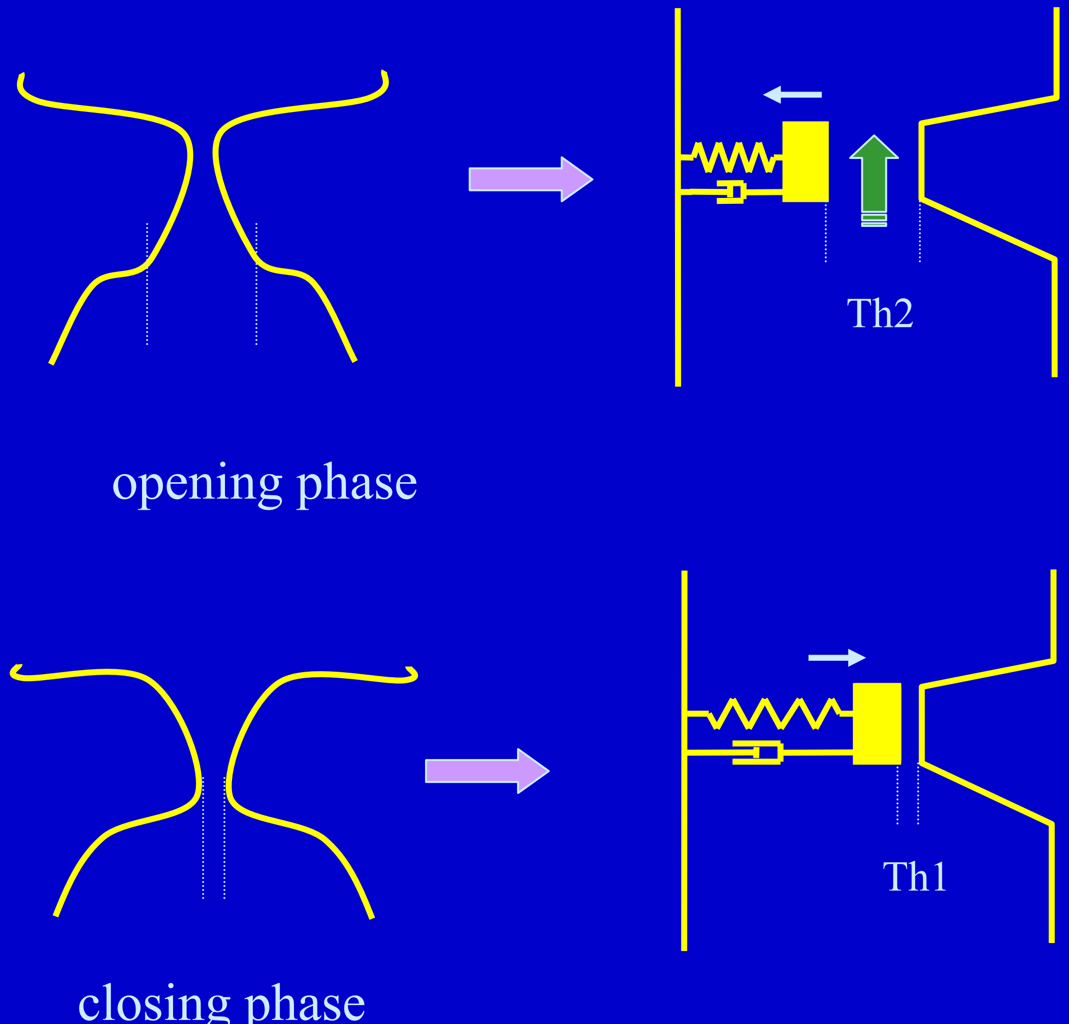
The physics of small-amplitude oscillation of the vocal folds. Ingo R. Titze. J. Acoust.Soc. Am.83(4), April 1988

Numerical simulation : simplifications



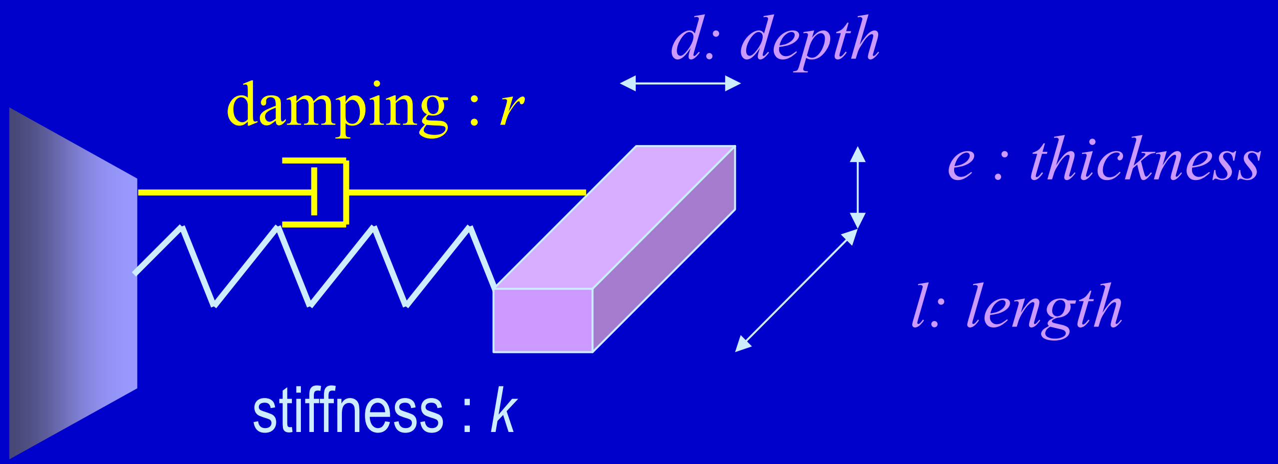


Simulation : Independent variables of the one degree of freedom model



- 1 spring
- 1 damping
- 1 mass
- The Bernoulli effect
- 2 thresholds of functioning
- Airflow supply

Parameter value of the one degree of freedom model of relaxator



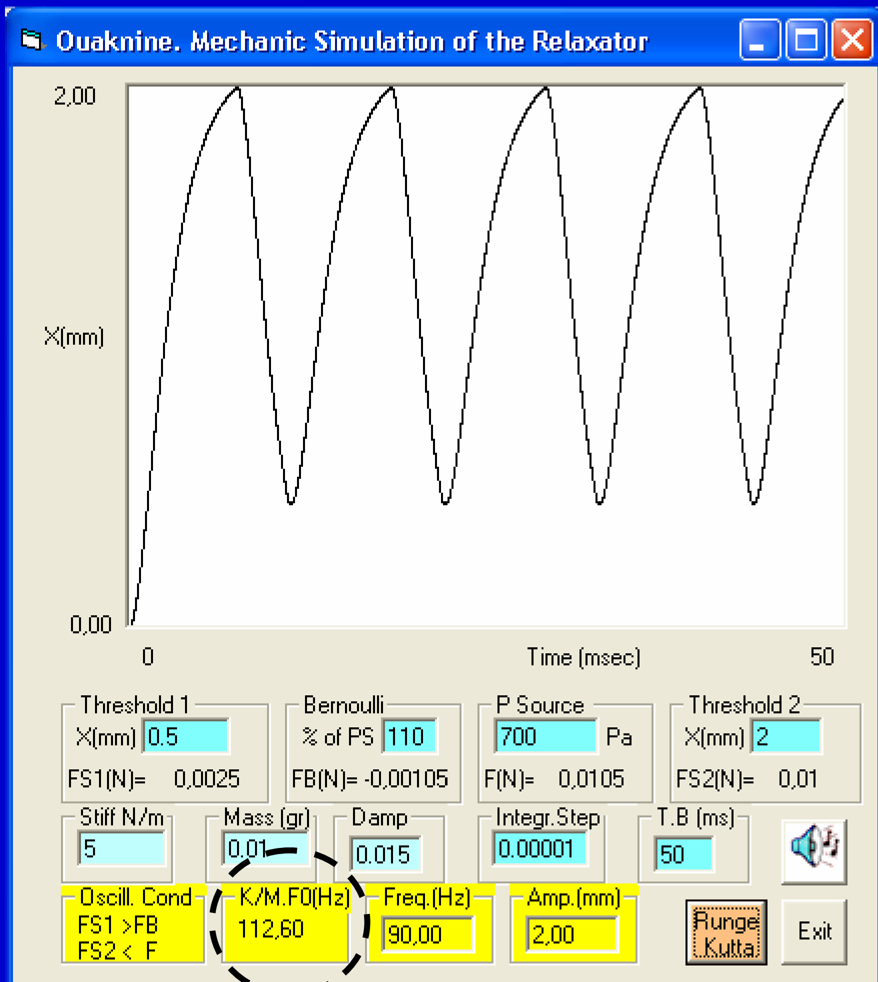
Spring stiffness : $k = 5.0$ N/m

Damping : $r = 0.015$ N.s/m

Surface of the vocal fold : $S = l \cdot e = 15.10^{-6}$ m^2

Mass : $m = 0.02$ g

Numerical simulation of a one mass model



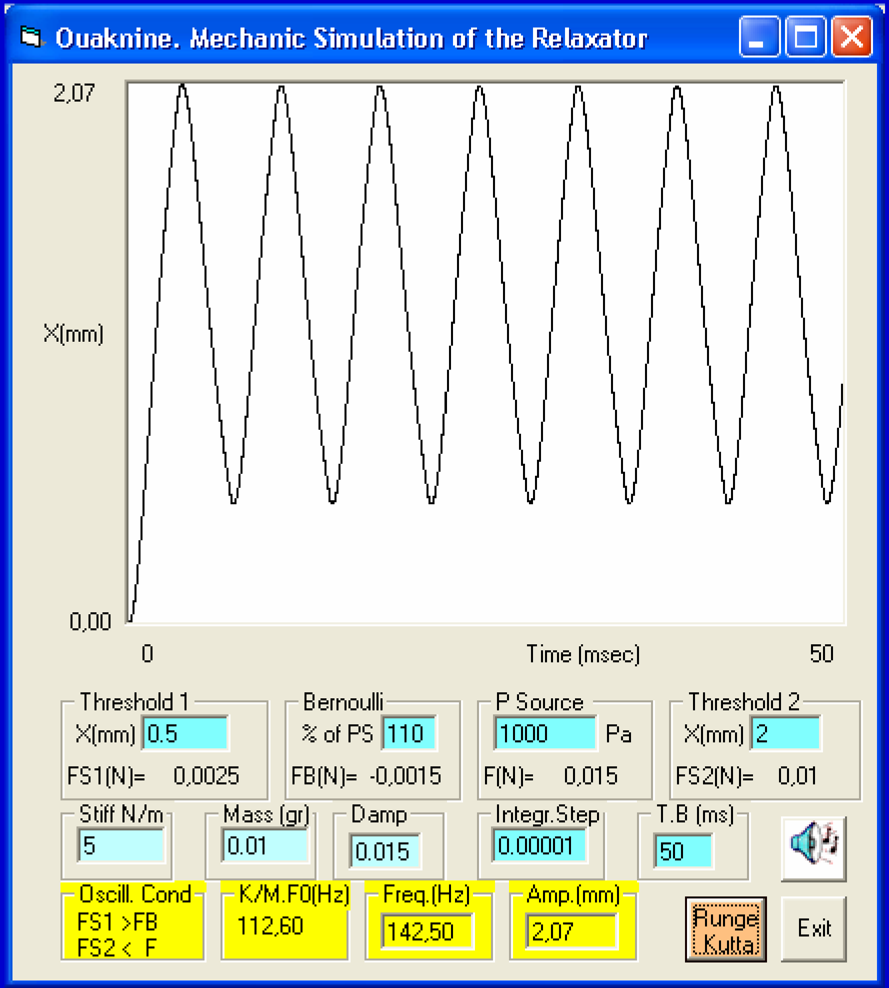
Let us apply a pressure source about 700 Pascal. The mass value is 0.01 g. The stiffness value is 5 N/m. The damping coefficient is 0.015.

Self oscillations are obtained. The waveform is of a relaxation type. The fundamental frequency is 90 Hz. The amplitude is 2 mm.

$$\frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Pendular relation

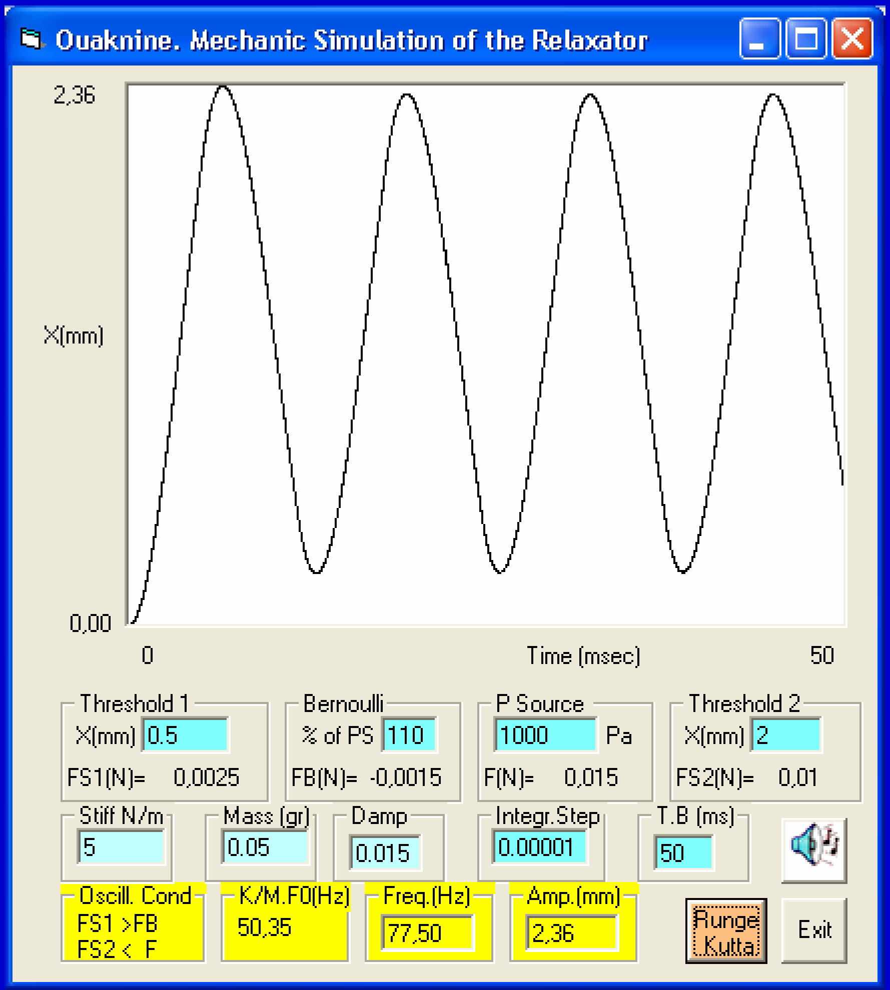
Numerical simulation of a one mass model



Let us increase the pulmonary pressure to 1000 Pa.

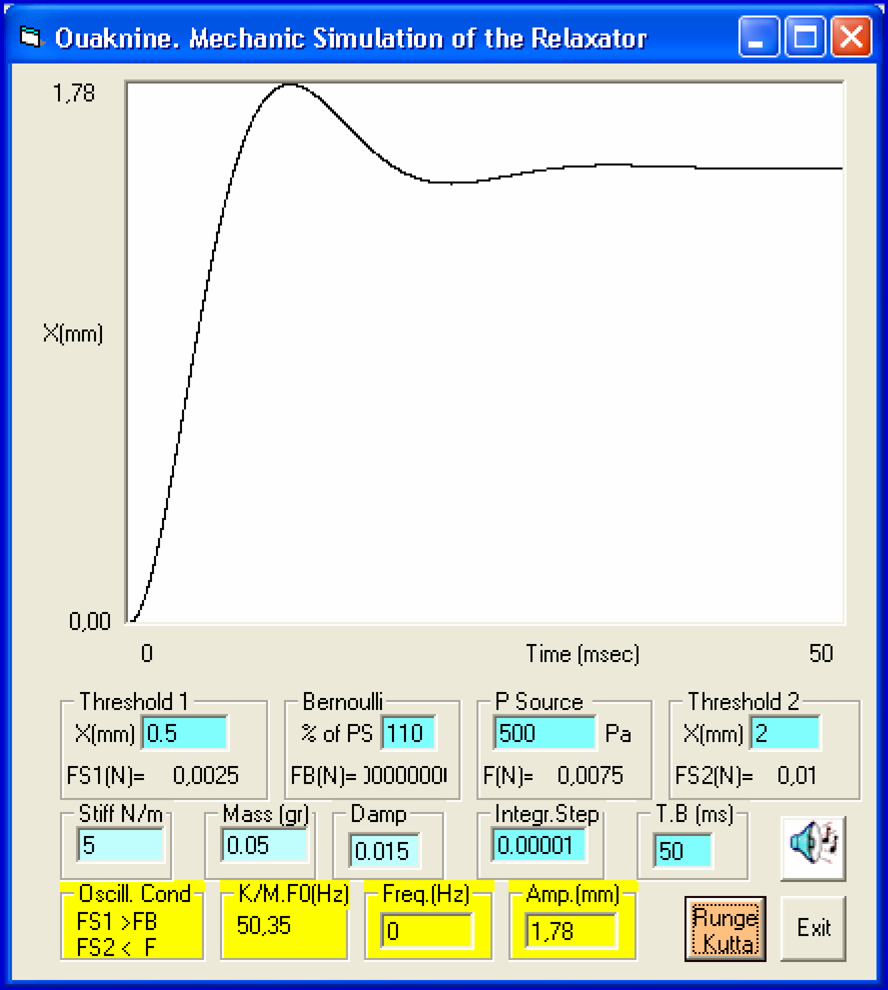
The frequency increased to 142.5 Hz and the amplitude to 2.07. The waveform is triangular shape.

Numerical simulation of a one mass model



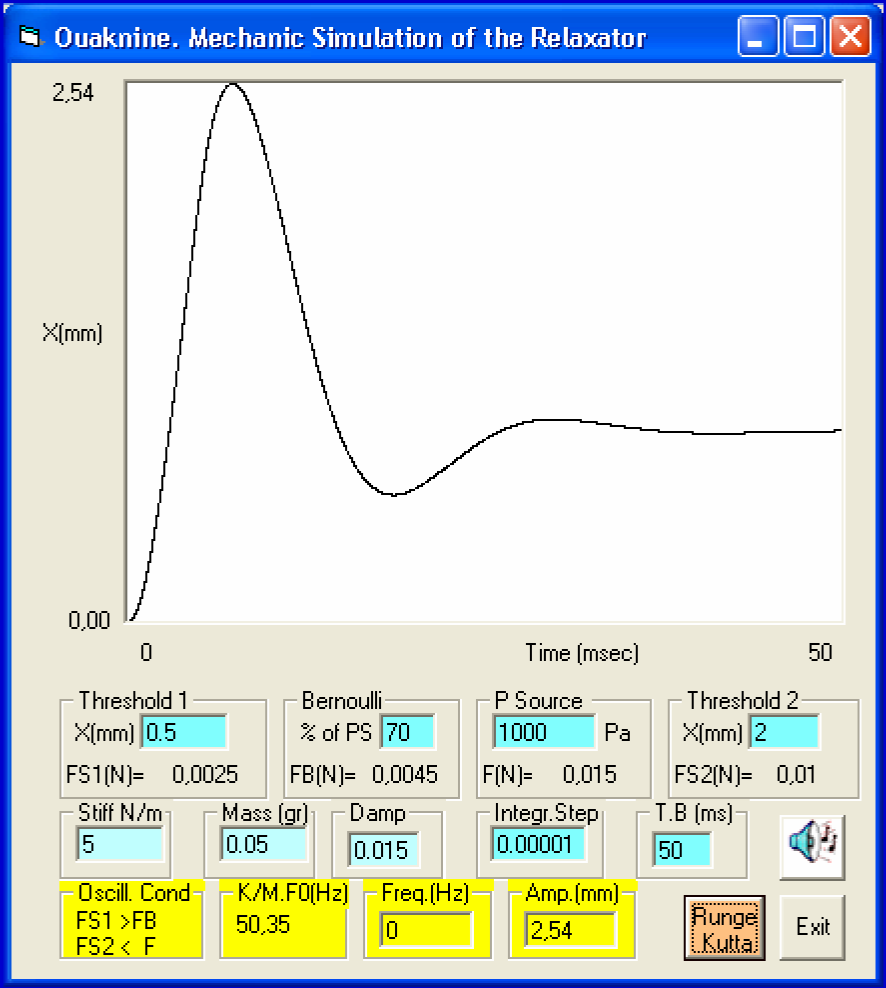
Now let us increase the value of the mass to 0.05 g. The waveform is more sinusoidal. The frequency decreases to 77.5 Hz and the amplitude increases to 2.36mm.

Numerical simulation of a one mass model



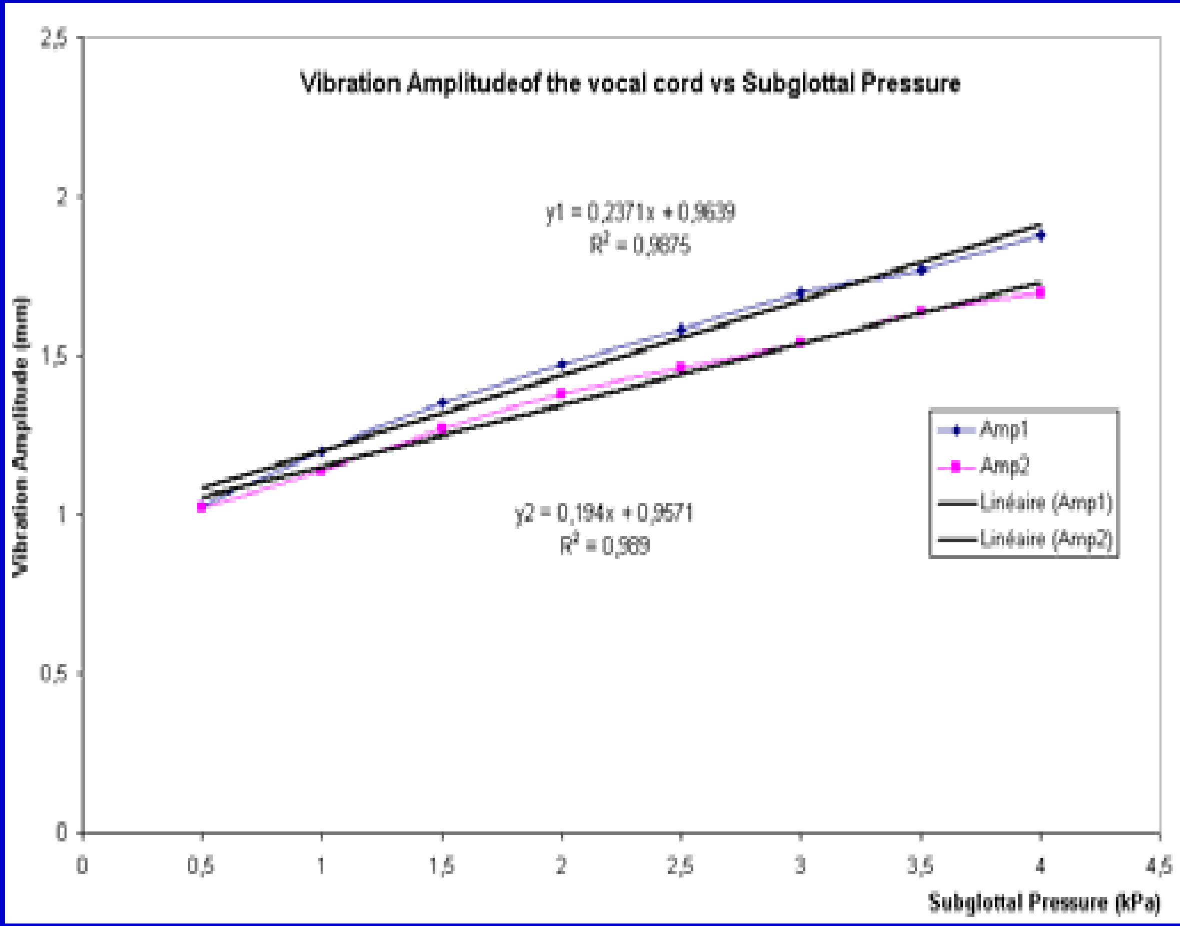
Now let us examine some conditions where the oscillations fail. First case, insufficient Pressure : 500 Pa. Opening threshold can not be reached.

Numerical simulation of a one mass model



Second case, insufficient Bernoulli effect to reach the closure threshold.

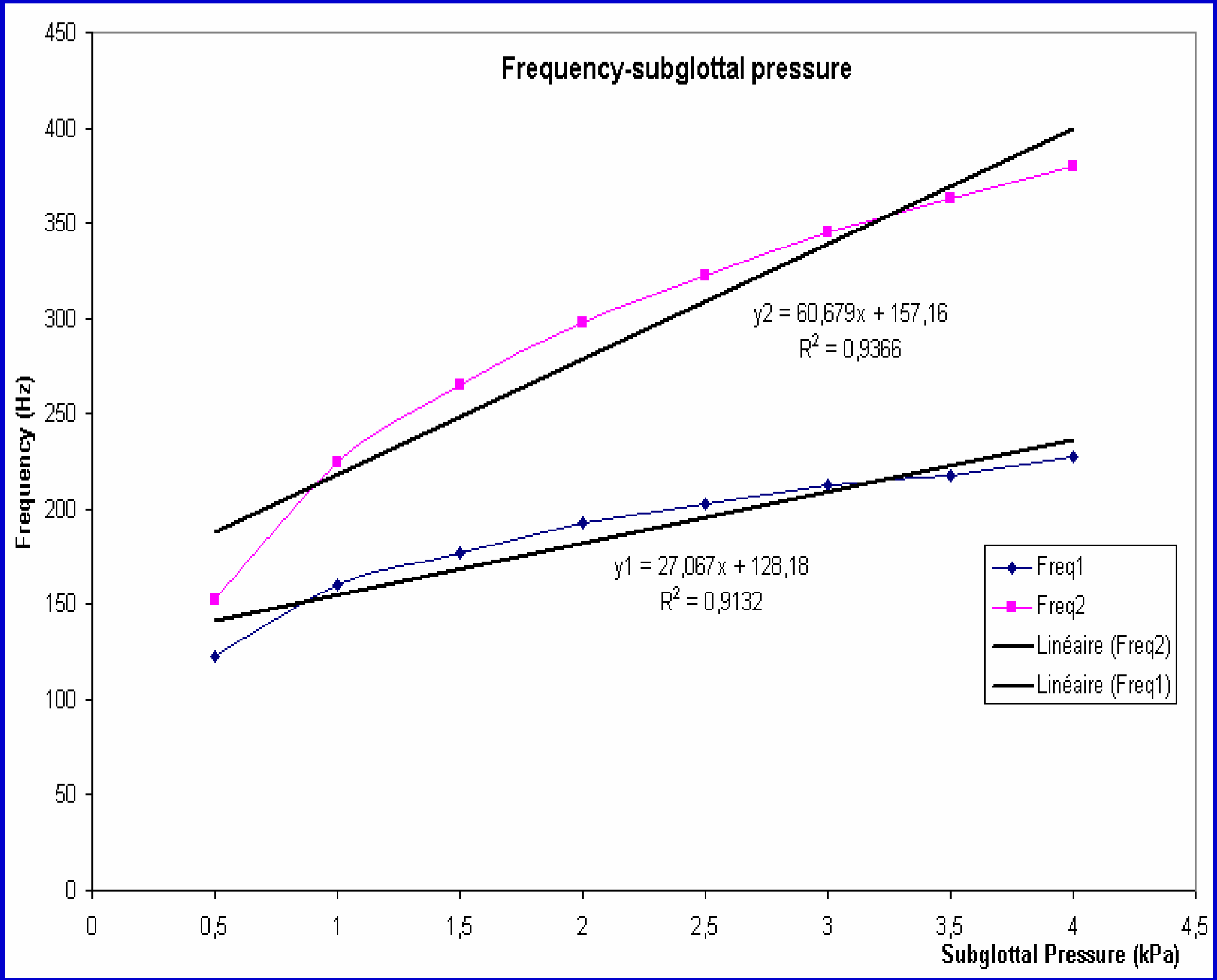
Vibration amplitude – Subglottal Pressure relation



Parameters

Th1 = 0.1 mm
Th2 = 1 mm
M = 0.01g
k = 5 N/m
r = 0.015 N.s/m
PB1 = -0.1xPSG
PB2 = -0.5xPSG

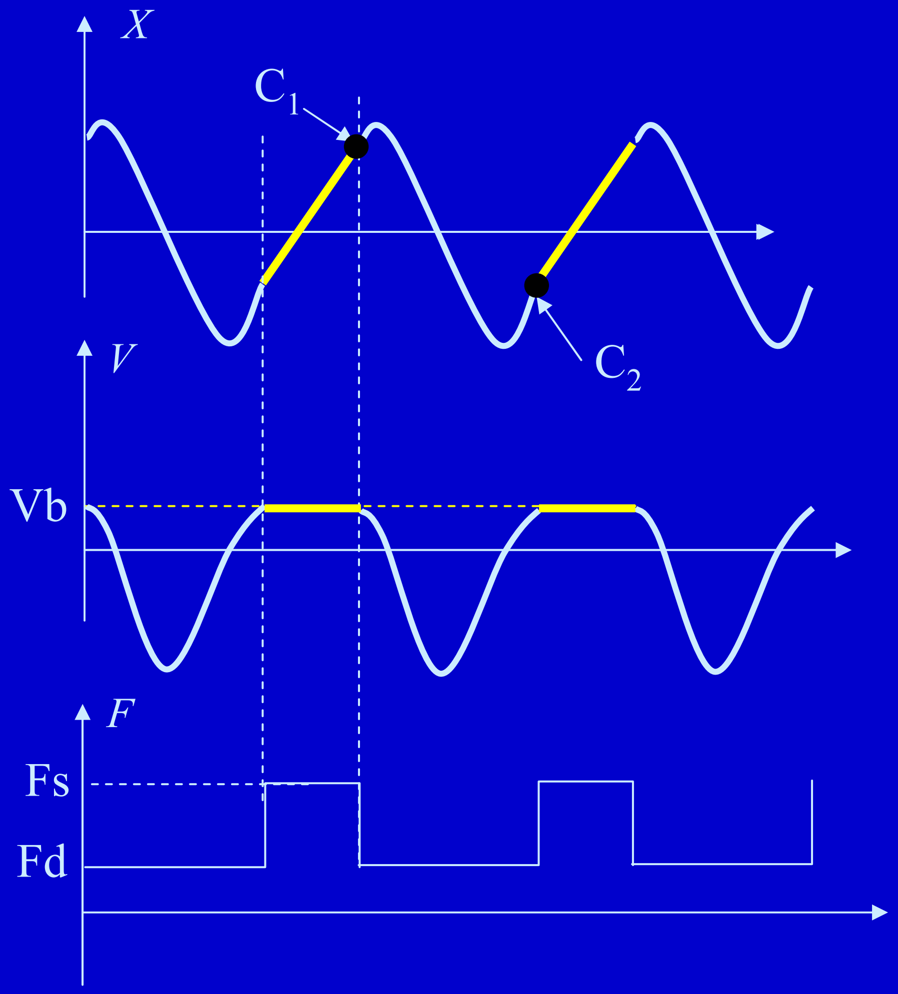
Frequency-subglottal pressure relation

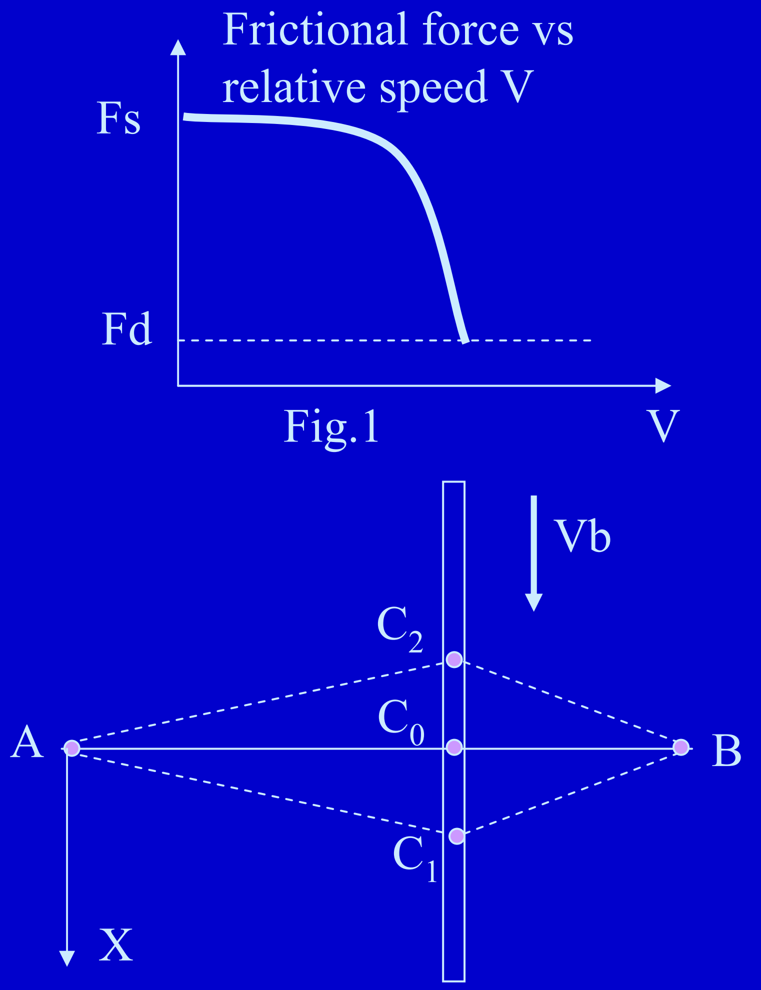


Parameters

Th1 = 0.1 mm
Th2 = 1 mm
M = 0.01g
k = 5 N/m
r = 0.015 N.s/m
PB1 = -0.1xPSG
PB2 = -0.5xPSG

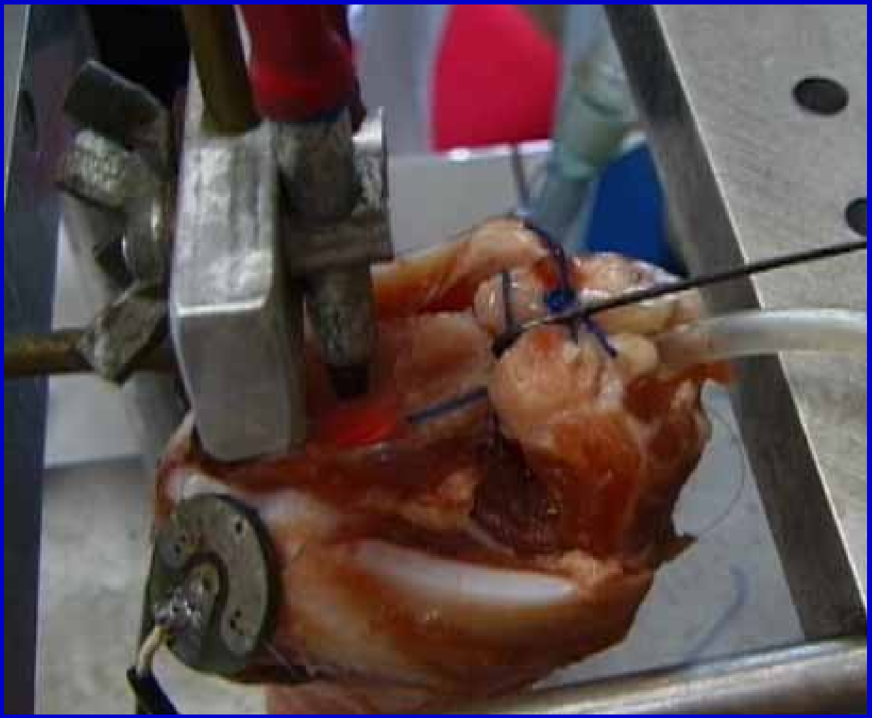
Bowed string model analogy : Stick and slip

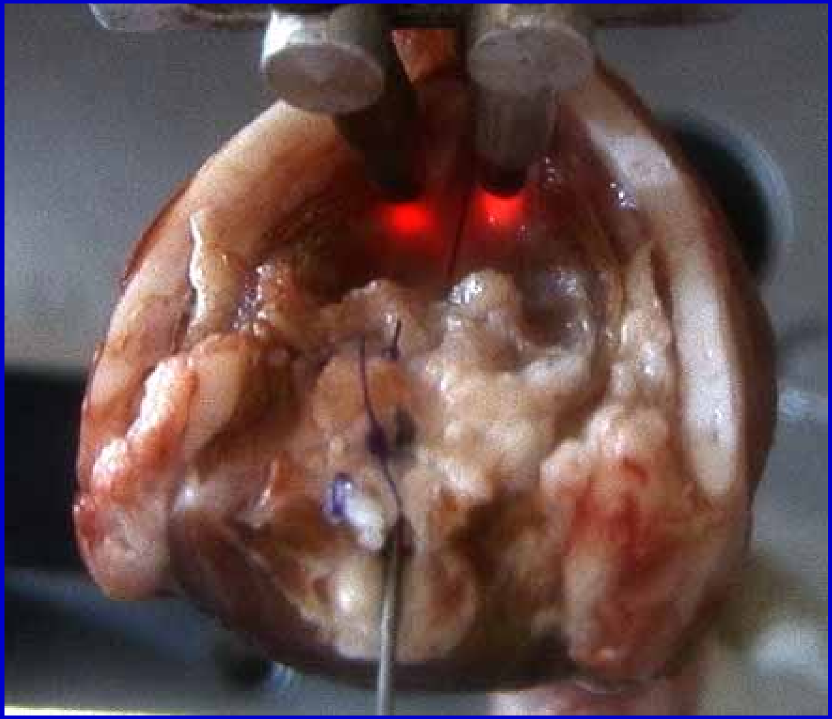




…But more complex is the reality…

Experimental observations (personal works): Set up





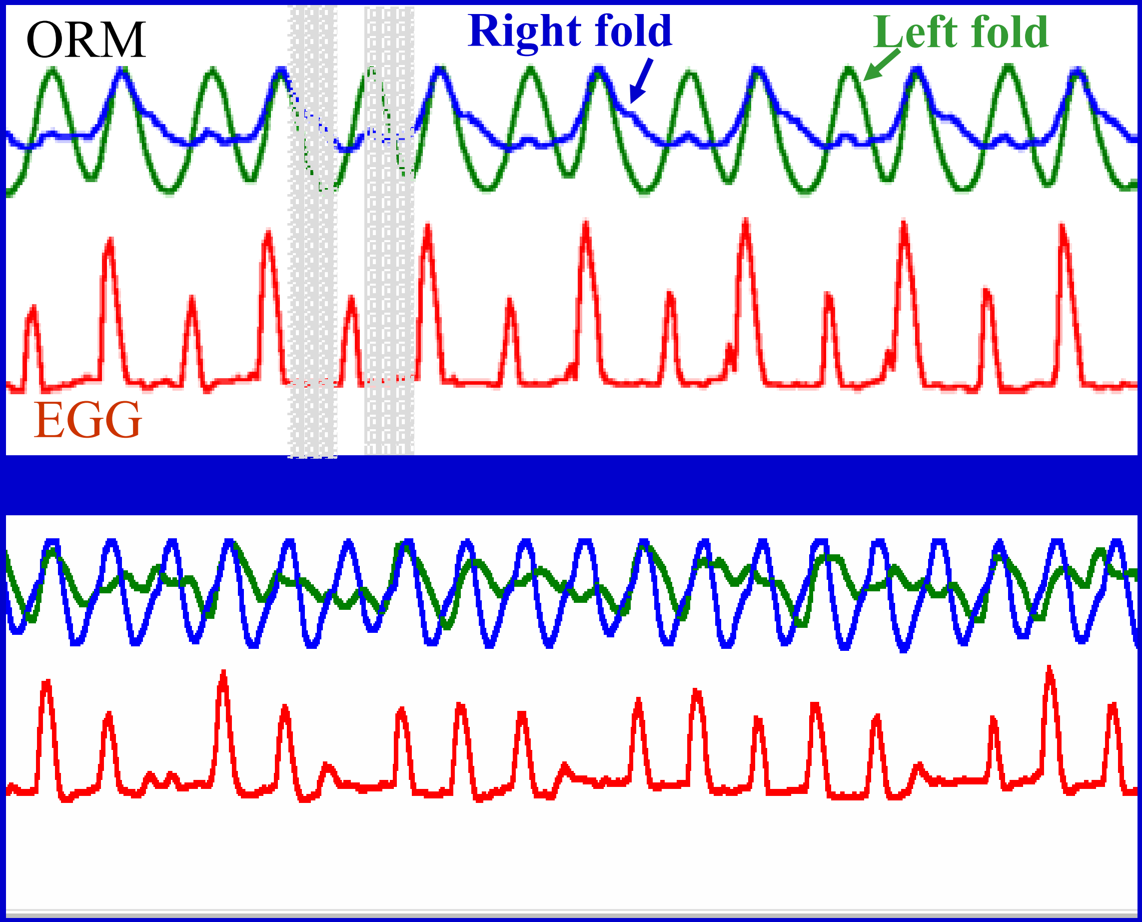
- Conditions of asymmetry
- Electroglottography (EGG)
- Optoreflectometry

2 vocal folds → Non linear interaction

Experimental observations: Results

Asynchronous vibration of the 2 folds

period doubling



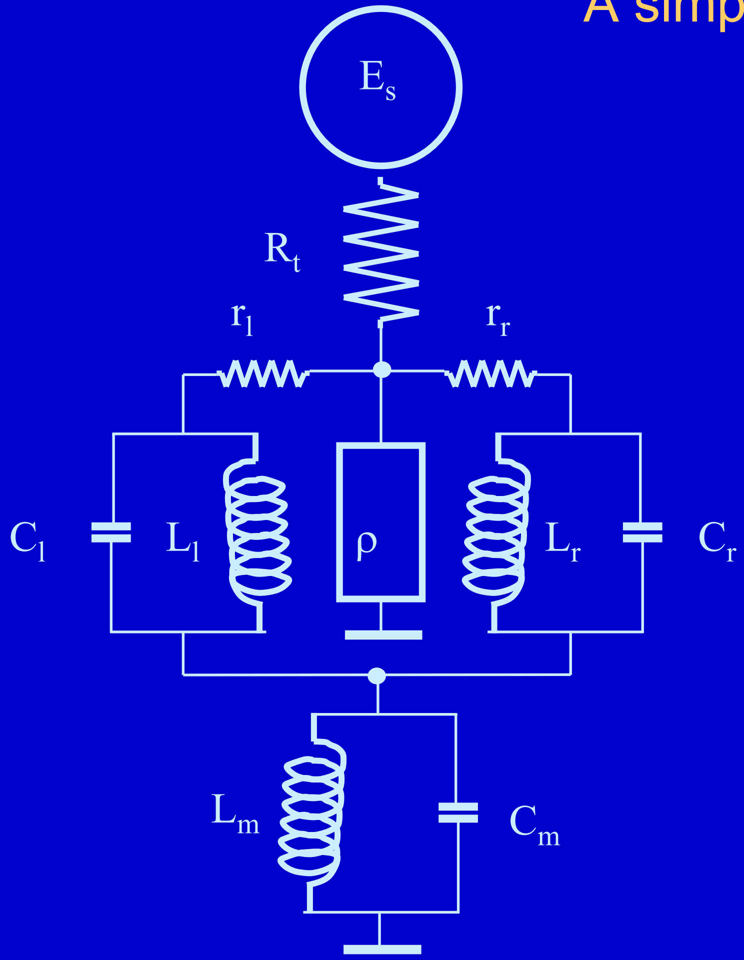
Chaos

Interaction of the vibration of the 2 folds.

MODELING : At least two non-linear coupled oscillators are needed

TWO COUPLED RELAXATORS

A simplified electric model



$E_s \Leftrightarrow$ Source pressure

$R_t \Leftrightarrow$ Tracheal resistance

$r_i \Leftrightarrow$ Resistance generated by vocal fold friction

$\rho \Leftrightarrow$ Non-linear glottic Impedance

$L_i \Leftrightarrow$ Fold mass

$C_i \Leftrightarrow$ Fold elasticity

$L_m \Leftrightarrow$ Shared mass

$C_m \Leftrightarrow$ Shared elasticity

M. OUAKNINE. Non-linear behavior of vocal fold vibration : Role of coupling.Advances in Quantitative Laryngoscopy, Voice and Speech Research 3rd International Workshop. Aachen june 19-20, 1998

Conclusions

- The model is consistent with main experimental studies about:
  - functioning threshold
  - dependence of amplitude an frequency on the subglottal pressure
- The conditions of self sustained oscillations are simple and clear

- The oscillation is made of the succession of different states. Each state can be describes by analytic equations. The transition between two successive states is abrupt.

Future studies

•The mathematical relation between the glottis deformation as function of the intraglottal pressure is needed to produce a more analytical model such as that obeiyng to the van der pol equations.

•The **vocal register** transition may be due to a drastic change of the values of the parameters (mass, stiffness, damping)

•The coupling between two relaxation oscillators may lead to non linear phenomenon such as **bifurcations and chaos**.

Addum

The simulation in Visual Basic can be obtain form the authors :

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