Modeling Coupled Aerodynamics and Vocal Folds Dynamics Using Immersed Boundary Methods*

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Abstract

The Immersed Boundary method, originally introduced by Peskin to model the function of the mammalian heart, is used to model the closely coupled dynamics of the vocal folds and the aerodynamics in phonation. Two-dimensional model vocal folds are simulated with material properties chosen which result in self-oscillations in physiological frequency ranges and with physiological volume flows. The vorticity field is studied in conjunction with the highly dynamic vocal folds motions, resulting in interesting correlations between them. Two cases of transglottal pressure gradient are discussed and a preliminary analysis of them is presented. Currently under development is a new approach to the Immersed Boundary methods using a high resolution Navier-Stokes solver, the vocal folds motion, and the methods of the parallelized Adaptive Mesh Refinement approach all working in concert.

1. Introduction

The modeling of the dynamics of phonation is a challenging system both from the standpoint of the inherent interdependence of glottal material properties, the motion of the vocal folds, and the complex aerodynamics which develops in the glottis. In this work we focus on simulations which utilize the Immersed Boundary (IB) methods developed originally by Peskin [1] and further elaborated by Peskin and McQueen [2] for the heart system. The IB methods treat the dynamics of the air and the vocal folds on a more or less equal footing. The emergent dynamics of the system comes from the coupled nature of the two interacting subsystems and provides a truly intrinsic characterization of the physics of phonation.

In the work we report here, we present some of the basic features of the IB methods, discuss the treatment of the material properties, and summarize the results of simulations using the combined system. The characterization of this self-oscillating complex system using IB methods allows one to follow and better understand the interplay between the vocal folds and the air and represents a novel approach to the treatment of the fundamental aspects of phonation. We present vocal fold dynamics snapshots and aspects of the vorticity dynamics generated by the vocal folds. The results indicate that this simplified two-dimensional model can yield interesting results which better inform us about the close interplay among the participating subsystems.

We also briefly describe some current research which uses a different approach to the coupled dynamics of the vocal folds and the air. In addition to more flexibility in boundary conditions, the new approach uses a high resolution Navier-Stokes solver in conjunction with the powerful method of parallel Adaptive Mesh Refinement. With such an approach we will be able to obtain much increased spatial and temporal resolution with the simulation laid on a massively parallel cluster. Using such methods, the move to three-dimensional system will be made much more practical. We discuss some present work along these lines.

2. Penalty immersed boundary (PIB) method applied to models of phonation

2.1 Summary of the PIB method

The basic idea of our approach to modeling the coupled air-vocal folds system is to treat the air as a viscous, incompressible fluid in which there is an immersed material boundary, here taken to model the vocal folds. The method handles the coupled system of air and vocal folds as a single, potentially strongly interacting whole. In general the coupling can involve quite complex motion of both the air and the vocal folds. The dynamics which emerges from this interaction can not be determined from approximations which assume one or the other of the subsystems dominates. In the physiological as well as computational model system the behavior must emerge from the complex interplay between the two subsystems. This is a strength of the PIB method that our work utilizes.

The equations which characterize the PIB method are given as follows: the air is assumed to satisfy the Navier-Stokes equations for a viscous, forced, incompressible fluid:

$$\rho\frac{\partial u}{\partial t} + \rho u \cdot \nabla u = -\nabla p + \mu\nabla^2 u + f \qquad (1)$$
$$\nabla \cdot u = 0$$

where ρ, u, p, and μ are respectively the density, velocity, pressure, and the viscosity of air. f is the force per unit volume exerted on the air by the vocal folds. The force f on the air due to the vocal folds is related to the material force F on the vocal folds due to internal forces in the material via

$$f(x,t) = \int F(s,t)\delta(x - X(s,t))ds \tag{2}$$

where δ is the Dirac delta function. The force F on a given material point is the sum of two parts: the force on the element at X due to adjacent elements, denoted by $S(X)$ which is determined by the material properties of the vocal folds, and the force on the boundary due to a spring connecting the massless boundary point labeled by X(s,t) to a particle of non-zero mass M residing at Y(s,t). This second massive particle attached to that at X is introduced in this PIB method to provide mass to boundary points indirectly through the separate points. These two points at X(s,t) and Y(s,t) only interact through a connecting spring. The boundary point which interacts with the air is characterized with the massless point at X, while this second massive particle Y is a device introduced to allow mass to be associated with the boundary while at the same time not requiring the boundary point itself to have mass.

This method has been introduced recently by Kim and Peskin [3] and has been demonstrated to serve in the desired manner, while at the same time allowing the usual IB method algorithm to be used essentially intact. The force on a boundary point due just to the massive particle at the end of the spring at the point Y(s,t) connected to the point of interest at X(s,t) is taken to be $F_m = -k(X - Y)$, where k is the spring constant. In general k is chosen so that the massive point at Y closely tracks the massless point at X. The total force on the boundary point is $F = S(X) + F_m$.

We impose the no-slip condition at the boundary point labeled by X. This is implemented as follows:

$$\frac{\partial X}{\partial t}(s,t) = u(X(s,t),t) = \int u(x,t)\delta(x - X(s,t)d^2x. \tag{3}$$

The motion of the massive particle with mass M at Y is governed by the equation of motion:

$$M\frac{\partial^2 Y}{\partial t^2} = -F_m \tag{4}$$

A single step forward in time is outlined as follows:(1) Given the position of all boundary points and the velocity field at time t, (2) compute the force F on each vocal fold point due to adjacent vocal folds points denoted generally by S(X), plus the spring force F_m exerted by the massive point on the massless boundary point X, (3) Spread this force to the fluid using Eq.(2), (4) solve the forced Navier-Stokes equations to find the velocity field at time t+dt, and use the no-slip condition Eq.(3) to advance the vocal folds points positions to time t+dt. Further, use Eq.(4) to advance the massive points at Y. This completes the step from t to t+dt.

2.2 Results of two-dimensional model studies

We have performed simulations using a simplified two-dimensional model of the vocal folds. In the PIB method the vocal folds are characterized using two sets of Lagrangian markers, one for each side. These particles move in response to forces exerted on them by the other, adjacent particles in the vocal fold material and exert a net force on the fluid. It is this force which appears in the forced Navier-Stokes equations. The forces between particles interior to the single layer of particles characterizing the vocal folds consist of stretching, bending, and fixed-point restorative types. The elastic constants describing the strength of these three kinds of force types determine the material properties, which in turn result in a net force acting on a given point interior to the vocal folds. These constants have been carefully chosen so that the resulting dynamics of the model exhibits behavior approximating known physiological characteristics. The particles move in response to the motion of the air surrounding them. The no-slip condition is used to update the particle positions dynamically. In addition we apply a body force over the first few rows of the computational domain with value chosen to induce the transglottal pressure gradient. The body force magnitude is chosen so that the induced transglottal pressure gradient is in the appropriate physiological range.

In the simulations we report here we have used an Eulerian grid of 256 x 4096 in the x- and y-directions respectively. The number of particles which make up the vocal folds depends on the constant Eulerian mesh spacing h in that the distance between adjacent particles must be no larger than h/2 to prevent leaking of the air through the boundary. We typically choose h/4 as that gives some extra coverage of the vocal folds. This can be useful when highly dynamic motions of the vocal folds occur in the course of the simulations. The Navier-Stokes solver makes efficient use of the Fast Fourier Transform to produce velocity and pressure fields which satisfy the incompressible, forced Navier-Stokes equations. In the present work we use a two-step, formally second order accurate solver[4].

As examples of simulations, we include two cases with each case corresponding to a different effective transglottal pressure gradient. The first case corresponds to a transglottal pressure gradient of ps=2 cmH_20 and the second to ps=8 cmH_2O. In the first case, the pressure gradient is insufficient to induce self-oscillation of the vocal folds. The second case is clearly above threshold for self-oscillation. In Fig. 1 we show a combined plot of the volume flow rates versus time, from which we can clearly see that the simulation method is sensitive to a threshold behavior of transglottal pressure gradient. Fig. 2 shows snapshots of a combined rendering of the vorticity field and the vocal folds for the sub-threshold case of 2 cmH_2O. Fig. 3 illustrates a sequence of vorticity and vocal folds position snapshots for the above-threshold case of 8 cmH_2O. The model allows the study of the relation between the dynamics of the vocal folds and the manner by which vorticity is produced and is moved by the self-oscillating vocal folds model. This will be a focus of future studies.

The results of these two cases illustrate the strength of the PIB method as applied to phonation. We have an intrinsically coupled model of the vocal folds-air system in which the emergent dynamics comes about from the close interplay between the forces exerted by the vocal folds on the air, and in turn the complex aerodynamics which results in vorticity production is made clearer. In further studies we will explore the connection between the role of vocal folds motion and vorticity production.

3. Parallel adaptive mesh refinement and its potential for capturing high resolution vocal folds dynamics and aerodynamics

One of the challenges of modeling the vocal folds using a fixed Eulerian mesh for the velocity and pressure fields with the Lagrangian particles moving on that mesh is to capture all the important physics in the model. As the Reynolds number effectively associated with the flow of air increases, the number of Eulerian mesh points needed to cover the physics increases. Even in two spatial dimensions it becomes clear that at some point the computational resources will be severely challenged to provide the needed resolution. In order to address this problem we have began a program to create an implementation of the IB methods in the context of parallel Adaptive Mesh Refinement technology coupled with a more general Navier-Stokes solver which can accommodate a variety of boundary conditions. Currently we are developing the two-dimensional PIB method using the infrastructure of methods developed at Lawrence Berkeley Labs (LBL) [5]. This computational technology has been developed in part to allow the implementation of a robust class of solvers for the forced Navier-Stokes equations for variable density systems. Underlying the solver is a set of meshes with varying degrees of resolution, with the meshes created as the physics being expressed requires more resolution. The meshes are dynamically created and destroyed as the simulation proceeds, giving needed coverage of small scale phenomena while doing so consistent with the physics at coarser scales. The net result of this approach will be to provide more accurate coverage of the physics of the vocal folds–air system and do so in a computationally efficient manner. In addition, this technology has been parallelized. This means that the evolution of the vocal folds system will be accomplished with much enhanced spatial and temporal resolution and done on large parallel clusters. This has an important practical payoff: by spreading the computations among a large number of processors, we can obtain much more resolution than with a simulation bound to just one processor. With these methods we expect that simulations will be feasible at previously practically unachievable levels of detail. We will report on the new approach in forthcoming papers.

4. Conclusions and future work

The results of applying the PIB method to simplified two-dimensional models of phonation show that the method does produce physiologically relevant properties. The method allows for the first time a truly intrinsic treatment of the complexities of the air-vocal folds system and demonstrates the close connection between the production of vortices by the dynamic vocal folds motion. The two subsystems are not separable in the real glottis and this method recognizes that inseparability and adapts to it. The success of our present experimental simulations indicates that we can now proceed to treat some types of pathologies and study the interesting relation between the occurrence of pathologies and their influence on the aerodynamics. Such work may result in a future method to predict measurable markers for some pathologies in the aerodynamics and perhaps be of clinical relevance. The acquisition of adequate resolution to capture important flow structures and vocal folds motion necessitates using methods which can dynamically obtain increased resolution as the simulation proceeds. This need can be met by using the parallel Adaptive Mesh Refinement technology pioneered at LBL by Almgren et al [5] and we are presently developing a new version of the PIB method in conjunction with that approach.

*Work supported in part by NIH NIH grant R01 DC03577 and in part by NIH grant R01 DC005788-01A1.

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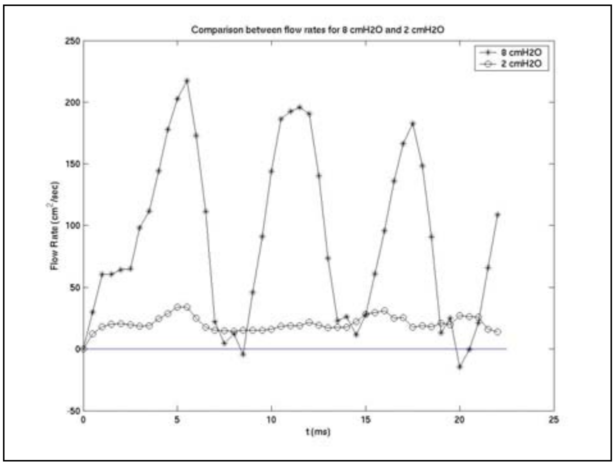

Figure 1 Volume flow for ps=2 cmH_2O and ps=8 cmH_2O

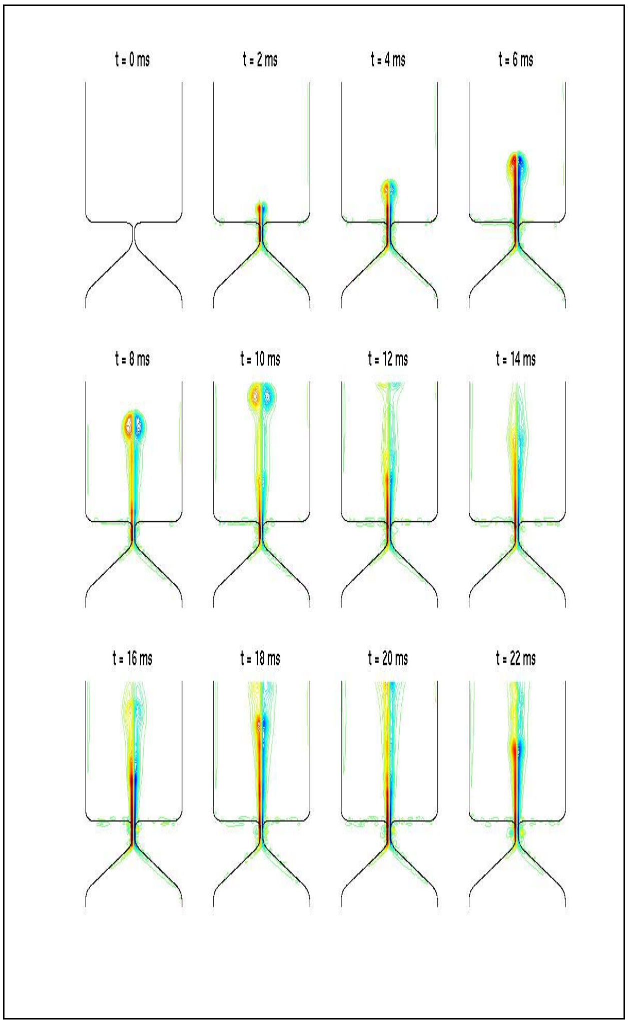

Figure 2 Vocal Folds and Vorticity Snapshots for ps=2 cmH_2O

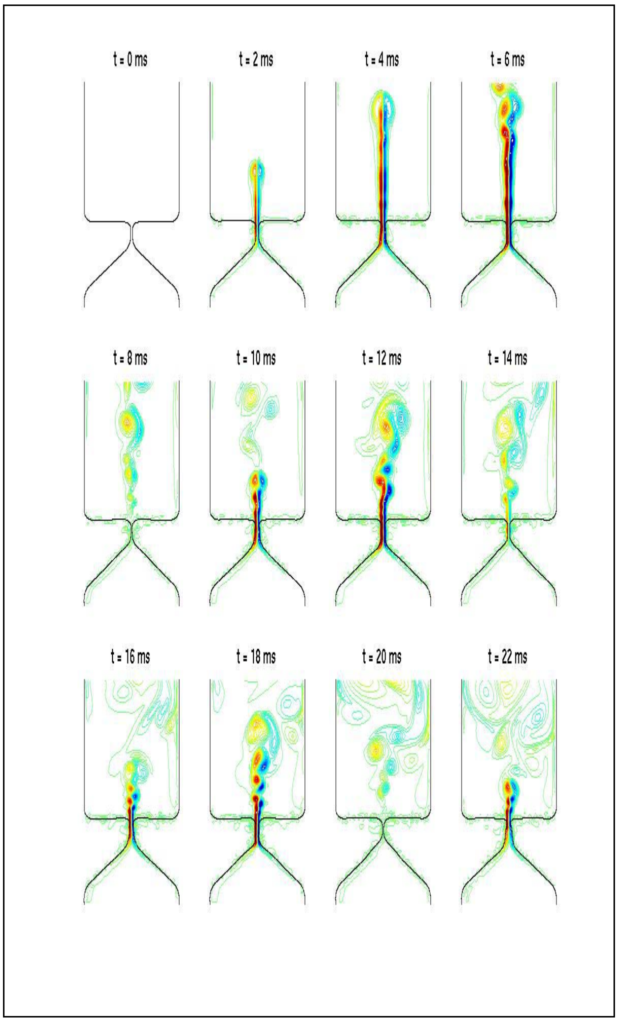

Figure 3 Vocal Folds and Vorticity Snapshots for ps=8 cmH_2O