

Complexity in birdsong production

G. B. Mindlin[†], R. Laje[†], D. Sciamarella[‡]

[†]Departamento de Física, Universidad de Buenos Aires, Argentina

[‡]LIMSI-CNRS, France

gabriel,rodrigo@df.uba.ar;sciamarella@limsi.fr

Abstract

The syrinx of oscine birds has two parts, each of which can be controlled, to some degree, independently by the bird. Beyond this independence, the resulting sounds when two sources are active can show evidence of coupling between them. In order to determine whether this kind of vocalization is in fact the result of the physics or the consequence of complex motor control, the complexity of the physics involved requires analysis beyond the cross modulation of the two signals. A preliminary study of the behavior of a nonlinear model of two sources acoustically coupled through the tract is presented.

1. Introduction

Until recently, it was assumed that the complex features of birdsong were directly controlled by the central nervous system [1]. A long research effort which investigated the motor activity of various muscles involved in the control of the syrinx [2], its anatomy and the nature of the sources of sound [3] has allowed to formulate adequate physical models of the syrinx [4] in order to close the path that leads from brain to sound. Recent modeling of the avian vocal organ, based on low-dimensional models proposed to generate voiced sounds in humans [5], shows that an appropriate mapping of the central neural activity onto peripheral motor action can be achieved through the study of correlations with specific control parameters (activity of syringeal muscles and respiratory pressure) instead of with the acoustic properties of the vocalizations [5,6,7]. In particular, rather complex sound can be produced when the two sound sources at the syrinx are active. So far, the existence of frequencies that were the sum and the difference the fundamental frequencies of the two sources was explained in terms of a generic nonlinear interaction [8]. In this work, we study the dynamics of a nonlinear syrinx model in order to explore the degree of complexity that the nonlinear mode interaction in the system involving the two sides of the bipartite organ adds to the song.

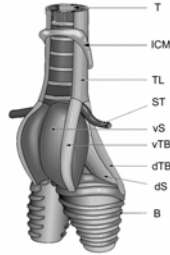


Figure 1: *The bipartite avian vocal organ.*

2. The model

Songbirds are capable of performing internal duets between their two sources of sound. The acoustic signals of one source can, under certain conditions, induce a change in the dynamics of the labia of the other sound source, as suggested by the data displayed in Figure 2. What are the mechanisms by which two coupled sound sources generate complex sounds?

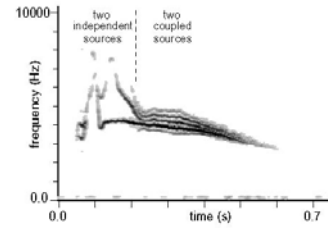


Figure 2: Sonogram.

To address this question, let us consider the dynamical system made up of the right ($i=r$) and left ($i=l$) sources:

$$\begin{aligned} \frac{dx_i}{dt} &= y_i \\ \frac{dy_i}{dt} &= f_i(x_i, y_i) + p_M^i(i, j) \\ f_i(x_i, y_i) &= -k_i x_i - b_i y_i - c_i x_i^2 y_i + d_i \end{aligned} \quad (2)$$

where x_i stands for the deviation of the rest position of the labia in source i , and d_i stands for the activities of the dorsal muscles. For further details on equation (2), see [9]. The average driving pressure p_M^i in each source is given by:

$$p_M^i(i, j) = p_L + \left(\frac{a_2}{a_1} \right)_i (p_L - p_I) \quad (3)$$

with :

$$p_I(i, j) = \alpha_i (x_{0i} + x_i - \tau_i y_i) + \beta_i (y_i - \tau_i \dot{y}_i) + \alpha_j (x_{0j} + x_j - \tau_j y_j) + \beta_j (y_j - \tau_j \dot{y}_j)$$

$$\left(\frac{a_2}{a_1} \right)_i = A_i - D_i y_i$$

where $i \neq j$, p_L is the lung pressure, p_I the tract input pressure, α_i and β_i the resistance and inertance coupling coefficients, τ_i the time the sound wave travels along the tract and a_1 , a_2 the channel entrance and exit areas (see figure 3). Notice that the effective driving pressure is a function of the geometry, which is assumed to depend linearly on labial velocity. Note also that p_I is common to both sources, hence introducing the acoustic coupling through the tract, which is made up of only one tube (we focus upon the interaction between sound sources rather

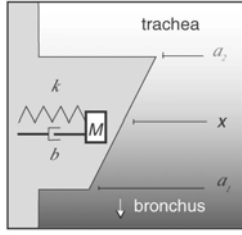


Figure 3: Mechanical model of the labium.

than on feedback effects). The acceleration terms in p_i can be rearranged so that the dynamical system takes the standard form: $\ddot{\bar{X}} = F(\bar{X})$. We will further assume that:

$$(D_1 y_1 - A_1) \beta_1 \tau_1 + (D_2 y_2 - A_2) \beta_2 \tau_2 \ll 1$$

Within this approximation, the equations for the labia read:

$$\begin{aligned} \frac{dx_i}{dt} &= y_i \\ \frac{dy_i}{dt} &= -\kappa_i x_i + \Pi_i y_i - c_i x_i^2 y_i - \varphi_i + \gamma_i x_j + \delta_i y_j + \\ &D_i [(\alpha_i \tau_i - \beta_i) y_i^2 + (\alpha_j \tau_j - \beta_j) y_i y_j - \alpha_i x_i y_i - \alpha_j x_j y_j] \end{aligned} \quad (4)$$

Assuming symmetry in $i=r, l$ for all control parameters except for κ_i ($\kappa_r \neq \kappa_l$) and Π_i , it can be shown that the linear part of the system has eigenvalues with zero real part at order three in the linear coupling coefficients γ and δ when $\Pi_r = -\Pi_l = 2\gamma\delta/(\kappa_r - \kappa_l)$. Making a linear transformation L to the base of the eigenvectors spanning this subspace, we compute the normal form which, for small γ and δ , can be conveniently written as:

$$\begin{aligned} \frac{dr_1}{dt'} &= r_1 (-\mu_1 + r_1^2 + B r_2^2) & \dot{\theta}_1 &= \omega_1 \\ \frac{dr_2}{dt'} &= r_2 (-\mu_2 + D r_2^2 + C r_1^2) & \dot{\theta}_2 &= \omega_2 \end{aligned} \quad (5)$$

with

$$\begin{aligned} r_1 &= A_1 |z_1|, r_2 = A_2 |z_3|, \begin{pmatrix} z_1 \\ z_3 \end{pmatrix} = \begin{pmatrix} \bar{z}_2 \\ \bar{z}_4 \end{pmatrix} = L \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \\ A_i(i, j) &= \left| \operatorname{Re} \left(-\frac{c}{2} \left[1 - \frac{\gamma^2 - 2i\gamma\delta\sqrt{\kappa_i} - \delta^2\kappa_j}{(\kappa_i - \kappa_j)^2} \right] \right) \right| |z_i| \end{aligned}$$

where μ_1, μ_2 are the unfolding parameters, $B, C, D, \omega_1, \omega_2$ are functions of the control parameters and $t' = -t$. Much of the behavior of (5) can be deduced by considering the family of planar vector fields obtained by ignoring the azimuthal components. It is easy to show that $(r_1, r_2) = (0, 0)$ is always an equilibrium and that up to three other equilibria can appear (in the positive quadrant):

$$\begin{aligned} (r_1, r_2) &= (\sqrt{\mu_1}, 0) \\ (r_1, r_2) &= (0, \sqrt{\mu_2/D}) \\ (r_1, r_2) &= \left(\sqrt{\frac{D\mu_1 - B\mu_3}{D - B C}}, \sqrt{\frac{\mu_3 - C\mu_1}{D - B C}} \right) \end{aligned}$$

Evaluating the control parameters at their typical values [9], one obtains $B < 0$, $C > 0$ and $D = 1$: the associated unfolding - in the (μ_1, μ_2) plane - is sketched in Figure 4, where the

bifurcation line with positive slope corresponds to $C\mu_1 = \mu_3$ and that of negative slope to $D\mu_1 = B\mu_3$. In order to seize the implications of these results for the unfolding of the four-dimensional flow $(r_1, r_2, \theta_1, \theta_2)$, the rotations $\dot{\theta} \approx \omega$ must be restored. The $(0, 0)$ equilibrium becomes a trivial fixed point, $(r_1 \neq 0, 0)$ is a periodic orbit with period $2\pi/\omega_1$, $(0, r_2 \neq 0)$ is a periodic orbit with period $2\pi/\omega_2$, and $(r_1 \neq 0, r_2 \neq 0)$ is an invariant two torus.

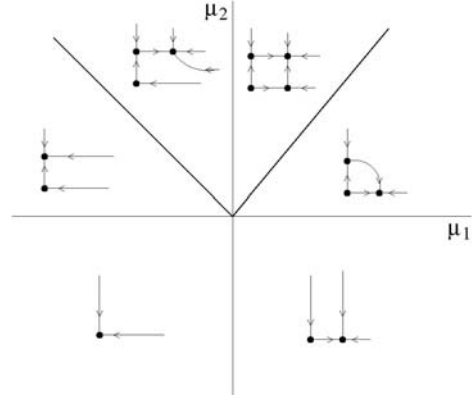


Figure 4: Planar (r_1, r_2) phase portraits of equation (5) for the unfolding when $B < 0$, $C > 0$ and $D = 1$.

3. Conclusion

We have considered a nonlinear model involving the two sides of the bipartite avian vocal organ coupled through the tract and analysed its properties in order to catalog mode interactions in parameter space when the labia tension in the right and left source is not symmetric. This analysis is consistent with numerical simulations, in which the activation of both modes in (5) translates into spectrally rich sonograms. These preliminary results suggest that nonlinear mode interaction involving the two sides of the avian vocal organ constitute an important factor in the production of spectrally complex syllables.

4. References

- [1] Konishi M., 1994. *Brain Behav. Evol.* 44 (4-5), 279-285.
- [2] Goller F., Suthers R.A., 1996. *J. Neurophysiol.* 76, 287-300.
- [3] Goller F., Larsen O. N., 1997. *Proc. Natl. Acad. Sci. USA* 94, 14787-14791.
- [4] Fee M.S., Shraiman B., Pesaran B., Mitra P. P., 1998. *Nature* 395, 67-71.
- [5] Titze I.R., 1988. *J. Acoust. Soc. Am.* 83 (4), 1536-1552.
- [6] Goller F., 1998. *Nature* 395, 11-12.
- [7] Suthers R.A., Margoliash D., 2002. *Curr. Opin. Neurobiol.* 12, 684-690.
- [8] Nowicki S., Capranica R. R., 1986. *Nature* 231, 1297-1299.
- [9] Gardner T., Cecchi G., Magnasco M., Laje R., Mindlin G.B., 2001. *Physical Review Letters* 87 art 1008101, 1-4.
- [10] Goller F., Suthers R.A., 1996. *J. Neurophysiol.* 75, 867-876.
- [11] Laje R., Mindlin G.B., 2002. *Phys. Rev. E* 65 art 051921.
- [12] Laje R., Mindlin G.B., 2002. *Physical Review Letters* 89 art 288102.
- [13] Mindlin G.B., Gardner T.J., Goller F., Suthers R., 2003. *Phys. Rev. E* submitted.
- [14] Laje R., Mindlin G.B., 2001. *Phys. Rev. E* 64 art 056201.