Representation of the Two-Dimensional Glottal Flow Using Irrotational and Rotational Flow Elements

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Abstract

This paper presents a method for modeling the two-dimensional glottal flow during the phonation. The glottis forms a diverging shape at the outlet and then vortices are produced from the thin boundary layer on the surface of the vocal folds, causing the pressure drop between the sub- and supra-glottal regions. Based on the polygonal line approximation of the glottal shape, the velocity field of the main flow region can be conveniently represented using the conformal mapping. Since the analytic function is known for the uniform flow and point vortex in the infinite strip domain, such flows in the physical domain can be calculated using the conformal mapping which connects both domains for specified glottal shape. In addition, the separation of the boundary layer is represented using the vortex method as the process in which a series of point vortices are released at the separating point. By taking the dynamics of these into account, the spreading of the vorticity from the edge of the glottis is represented as the movements of the generated vortices.

1. Introduction

To understand the nature of sound source mechanism of speech as the interactive process of the flow and vocal folds tissue, it is quite important to evaluate the flow through the glottis. During the phonation, it is assumable that the irrotational, uniform flow is supplied from the lungs. Rotational flow may also be produced from the boundary layer separation when the glottis takes a diverging configuration. Such flow component is essential in the production of glottal waves since the pressure loss is induced due to vortex-related momentum change and viscous dissipation.

To predict the velocity field according to the glottal shape and flow characteristics, this paper presents a two-dimensional flow model passing through the coronal section of the glottis. When the flow is incompressible and inviscid, the two-dimensional flow can generally be represented in a convenient manner using the complex velocity potential [1]. Also, flow pattern through a bounded region can be represented by the method based on the conformal mapping. By adopting the polygonal approximation of the glottal shape, this physical domain is connected analytically to a canonical domain formed as the infinite strip. In the canonical domain, general form of the potential function is known for several types of flow elements such as the uniform flow (irrotational element) and point vortex (rotational element) and hence the flow pattern in the physical domain can be derived by the connecting mapping function based on the Schwarz-Christoffel formula [2]. The generation of the vorticity can be expressed using the same framework by appropriately placing the point vortices into the main flow region based on the vortex method [3].

This paper is organized as follows. Section 2 mathematically explains the flow model in terms of the conformal mapping and velocity potential. Section 3 explains the generation and convection of the vorticity using the same framework. Section 4 presents simulation results based on the proposed flow model and Section 5 summarizes our work and gives the conclusions.

2. Representation of the two-dimensional glottal flow

This section describes the method for modeling the two-dimensional glottal flow. The glottal flow typically has a relatively small Mach number if the channel is not so constricted. In addition, the Reynolds number is of the order of 1000 [4]. Therefore, the incompressible and inviscid assumptions can be applied to the present problem. In our model, the boundary shape of the glottis is approximated using polygonal lines as shown in Fig. 1. This physical domain is taken in the extended complex z-plane bounded by lines with vertices $z_1,\ldots,z_{18}$. The left and right hand sides of the figure connect to the tracheal and pharyngeal regions, respectively. Fig. 2 illustrates the infinite strip domain taken as the canonical domain of the conformal mapping.

2.1. Complex variable representation of the flow field

For the two-dimensional incompressible flow, the continuity equation implies that the divergence of the velocity vector $\boldsymbol{v}=(u,v)$ is zero:

$$\nabla \cdot \boldsymbol{v} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{1}$$

If the flow is irrotational, the velocity component (u,v) can be represented using a velocity potential ϕ and stream function φ as

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial \varphi}{\partial y}, \quad v = \frac{\partial \phi}{\partial y} = -\frac{\partial \varphi}{\partial x}. \tag{2}$$

This forms the Cauchy-Riemann relation for the real and imaginary parts of the following complex function

$$f(z) = \phi(x,y) + i\varphi(x,y), \tag{3}$$

where $i=\sqrt{-1}$ and $z=x+iy$. Considering Eq. (2), derivative form of Eq. (3) gives the velocity compotents (*u,v*) as

$$\frac{df}{dz}=u-iv. \tag{4}$$

This gives the complex velocity $w=u-iv$. For this reason, *f*(*z*) is called the complex potential function.

2.2. Construction of the potential function via the conformal mapping

Eq. (4) indicates that the velocity field in the physical domain can be represented if the potential function *f*(*z*) is known. However, it is quite difficult to derive it directly for the given geometrical configuration of the glottis. To solve the problem, it is possible to adopt the method based on the conformal mapping where the physical domain of the flow field is connected analytically to the canonical domain via the mapping function. For the infinite strip canonical domain shown in Fig. 2, we can derive the potential function explicitly for a number of typical flows as will be discussed in the next subsection. Thus, the flow pattern in the physical domain can be obtained from these complex potentials and the connecting mapping function.

When the glottal shape is approximated using the polygonal lines, the conformal mapping function can be expressed based on the Schwarz-Christoffel formula as

$$z=g(\zeta)=C\int^{\zeta}\prod_{p=1}^{N}\sinh\frac{\pi}{2}(\zeta-\zeta_p)^{\frac{-\beta_p}{\pi}}\,d\zeta, \tag{5}$$

where C is a complex scaling factor of the mapping, N is the number of vertices and β_p is the prevertex corresponding to z_p as $\zeta_p=g^{-1}(z_p)$. The location of the prevertex ζ_p is determined numerically using enforcing conditions on the side length of polygonal lines. Given the potential function $f(\zeta)$ in the canonical domain and the mapping function $z=g(\zeta)$, the complex velocity $w=u-iv$ in the physical domain can be derived as

$$w=\frac{df}{dz}=\frac{df}{d\zeta}\frac{d\zeta}{dz}=\frac{df}{d\zeta}\frac{1}{g'(\zeta)}. \tag{6}$$

2.3. Potential functions in the infinite strip domain

This subsection describes potential functions representing the uniform flow and rotational flow due to a point vortex in the canonical domain. We can describe the entire flow pattern by overlapping these flow elements as shown in the next section. The potential function is derived so that the boundary condition

$$\nabla\phi(\xi,\eta)\cdot\boldsymbol{n}=0 \tag{7}$$

is satisfied, where $\boldsymbol{n}$ is a unit vector normal to the boundary of the infinite strip.

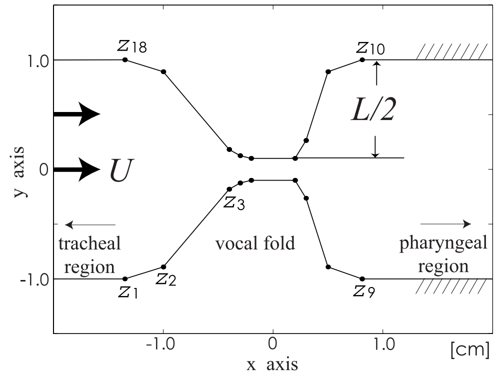

Figure 1: *The actual flow field is represented in the physical domain. The boundary shape of the glottis is flexibly approximated by polygonal lines in the z-plane.*

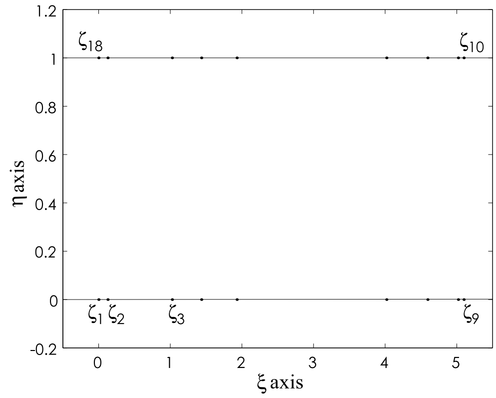

Figure 2: *Infinite strip in the ζ-plane is used as the canonical domain of the conformal mapping.*

2.3.1. *Uniform flow (irrotational element)*

The uniform flow can represent the expiratory flow from the lungs. The analytic function which describes the uniform flow is given as

$$f_u=U\zeta. \tag{8}$$

From $df_u/d\zeta=U$, this function clearly represents the flow U going from left to right along the ξ-axis of Fig. 2. It is easy to confirm that this flow satisfies the boundary condition given in Eq. (7).

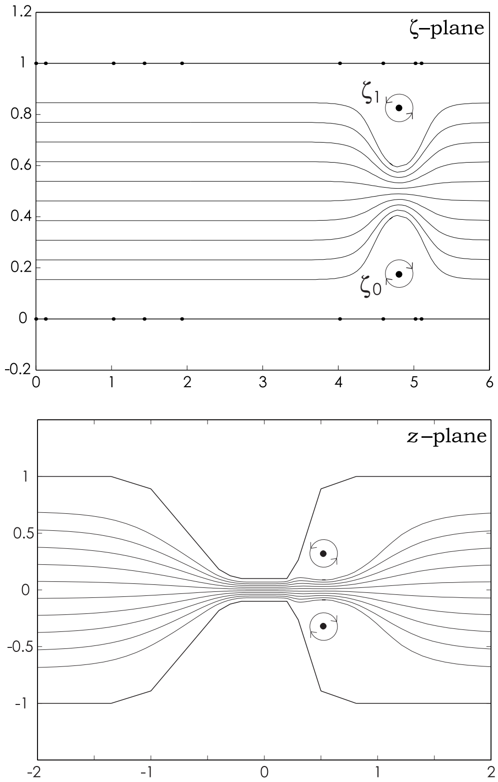

Figure 3: *Streamline of the glottal flow with a vortex pair in the infinite strip domain (top) and in the physical domain (bottom).*

2.3.2. Flow due to a point vortex (rotational element)

Though the influence of the viscosity is small in the bulk of the glottal flow, it cannot be neglected on the surface of the glottal wall, namely, boundary layer. It separates from the glottis at the point where the glottis takes a diverging configuration and produces a vortical flow in the main flow region. Therefore, the potential function should be derived for the point vortex when the boundary layer separation is to be considered following the vortex method.

Flow due to the point vortex is represented in the canonical domain by the following analytic function

$$f_{vi} = -i\frac{\Gamma_i}{2\pi}\log\frac{\sinh\frac{\pi}{2}(\zeta-\zeta_i)}{\sinh\frac{\pi}{2}(\zeta-\overline{\zeta_i})}. \tag{9}$$

The derivative form of this equation is expressed as

$$\begin{aligned}\frac{df_{vi}}{d\zeta} &= \Gamma_i\left[-i\frac{1}{4}\left\{\coth\frac{\pi}{2}(\zeta-\zeta_i)-\coth\frac{\pi}{2}(\zeta-\overline{\zeta_i})\right\}\right] \\ &= \Gamma_i K(\zeta,\zeta_i),\end{aligned} \tag{10}$$

where K is called the velocity kernel. The flow from this function is irrotational everywhere except at the point where the vorticity is placed: i.e., the circulation Γ_i is concentrated at the point $\zeta=\zeta_i$. Also, it satisfies the boundary condition given in Eq. (7). Explanations for deriving this potential function can be found in [5].

2.3.3. Streamline of the uniform flow with a vortex pair

Once the irrotational and rotational elements are determined in the canonical domain, the velocity field in the physical domain can be evaluated using the relationship given in Eq. (6). For instance, the potential function which describes the glottal flow with a vortex pair can be represented in the infinite strip domain as the sum of Eqs. (8) and (9) as

$$f = U\zeta - \frac{\Gamma}{2\pi}\log\frac{\sinh\frac{\pi}{2}(\zeta-\zeta_0)\sinh\frac{\pi}{2}(\zeta-\overline{\zeta_1})}{\sinh\frac{\pi}{2}(\zeta-\overline{\zeta_0})\sinh\frac{\pi}{2}(\zeta-\zeta_1)}. \tag{11}$$

The first term means the uniform flow supplied from the lungs and the second one a vortex pair of opposite circulation.

Fig. 3 shows the streamline formed by the potential functions in Eq. (11) and the mapping function in Eq. (5). The top and bottom of the figure correspond to calculated results for the canonical and physical domains, respectively. This simply simulates the glottal flow just after the separation of the boundary layer and the formation of the vortex pair. Detailed representation of such phenomena will be discussed in the next section using the vortex method.

3. Modeling the generation and convection of vorticity using the flow elements

The boundary layer separation from the surface of the glottis is schematically shown in Fig. 4. In this figure, we employ a curvilinear coordinate (ξ,η) which is adapted to the shape of the glottal wall. $u=u(\xi,\eta)$ and $U_0=U_0(\xi,\eta)$ are the velocity inside and outside of the boundary layer, respectively. The flow can be assumed to be inviscid outside the layer while the normal derivative of the velocity $du/d\eta$ is quite large inside and hence the viscosity is essential. In a divergent glottal channel, the flow is decelerated due to the positive pressure gradient and this velocity gradient becomes zero at a point as

$$\left(\frac{du}{d\eta}\right)_0 = 0. \tag{12}$$

This is the criterion of the separation point of the boundary layer where the vorticity is generated and released to the main flow region. Then the vorticity is convected and diffused according to the vorticity equation of motion. This section describes a model to this phenomena based on the vortex method.

3.1. Representation of the boundary layer separation

This subsection describes the modeling method of the boundary layer separation. In the vortex method, the boundary layer separation is represented so that point vortices are released at every moment from the separation point. The core function is applied to each vortex to express the spatial continuity and spreading of the vorticity [6].

Using the boundary layer approximation theory, for instance, based on the method of H. Holstein and T. Bohlen [7], we can calculate the separation point $z_{sep} = g(\zeta_{sep})$ and thickness δ of the boundary layer. The potential velocity U_0 can be derived using the complex velocity w in Eq. (6) as

$$U_0 = \left|\overline{w}\right| = \left|\overline{\frac{df}{d\zeta}\frac{1}{g'(\zeta)}}\right|. \tag{13}$$

The boundary layer can be assumed to move at the mean speed of the potential velocity U_0 and the velocity on the surface of the glottal wall. Therefore, the releasing point z_{rel} and the circulation Γ of the point vortex which is separated at the point z_{sep} are respectively determined using the first-order finite differences as

$$z_{rel} = z_{sep} + \frac{1}{2}\overline{w}\Delta t \tag{14}$$

and

$$\Gamma = -\frac{1}{2}U_0{}^2\Delta t, \tag{15}$$

where Δt is the time step of the simulation.

When the number of released point vortices is n, the vorticity in the physical domain can be approximated in the following form

$$\omega \approx \sum_{i=1}^{n}\Gamma_i f_\sigma(z - z_i), \tag{16}$$

where $z_i = g(\zeta_i)$ is the position of the point vortex and $f_\sigma(z)$ is the core function which spatially distributes each point vortex. In this study, we employ the gaussian core function as

$$f_\sigma(z) = \frac{1}{2\pi\sigma^2}\exp\left(\frac{-|z|^2}{2\sigma^2}\right), \tag{17}$$

where σ is the radius determined to be the half of the boundary layer thickness δ. Also, it satisfies the condition:

$$\int^z f_\sigma(z)dz = 1 \tag{18}$$

which means that the core function keeps the total circulation of the present flow. This is the necessary condition for satisfying the vorticity conservation laws that can be derived from the vorticity equation of motion.

3.2. Modeling the convective and diffusing motion of the vorticity

When the vorticity is released to the main flow region, it is well known that its motion is governed by the vorticity equation of motion. The two-dimensional vorticity equation of motion is formed as

$$\frac{D\omega}{Dt} = \nu\nabla^2\omega, \tag{19}$$

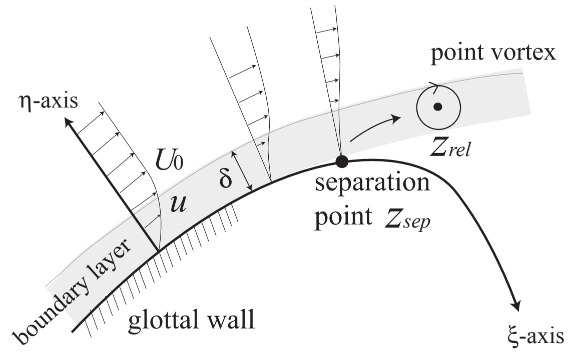

Figure 4: *The boundary layer separation from the surface of the glottis and its model representation using the vortex method.*

where ν is a dynamic coefficient of viscosity. This equation expresses that the vorticity is convected and diffused due to the viscosity. In Eq. (16), vorticity ω is approximately expressed as the sum of point vortices with the core function. Therefore, the process for solving the governing equation in Eq. (19) is split into the convection step of the point vortices and the diffusion step of the core radius of each point vortex according to the diffusion equation [8].

3.2.1. *Convection step of the point vortices*

Suppose that n point vortices are released in the physical domain. Considering the self-induced velocity, the convective velocity of the i-th point vortex is expressed as follows

$$\frac{dz_i}{dt} = \left[\sum_{j=1,j\neq i}^{n}\Gamma_j K_\sigma(\zeta_i,\zeta_j) - i\frac{1}{4}\Gamma_i\left\{-\coth\frac{\pi}{2}(\zeta_i - \overline{\zeta_i})\right\}\right]\frac{1}{g'(\zeta_i)}, \tag{20}$$

where z_i, ζ_i and Γ_i are the position in the physical domain, position in the infinite strip domain, and circulation of the i-th point vortex, respectively. $g'(\zeta)$ is the derivative of the mapping function shown in Eq. (5). K_σ is the convolution of the velocity kernel K and the core function f_σ as

$$K_\sigma(\zeta_i,\zeta_j) = K * f_\sigma = K(\zeta_i,\zeta_j)\left(1 - \exp\frac{-\left|\zeta_i - \zeta_j\right|^2}{2\sigma^2}\right), \tag{21}$$

where K and f_σ are shown in Eqs. (10) and (17), respectively. In Eq. (20), the first term in brackets means the velocity induced by the other point vortices and second one the self-induced velocity from its image. The convection of the i-th point vortex can be evaluated by integrating the ordinary differential equation in Eq. (20) numerically.

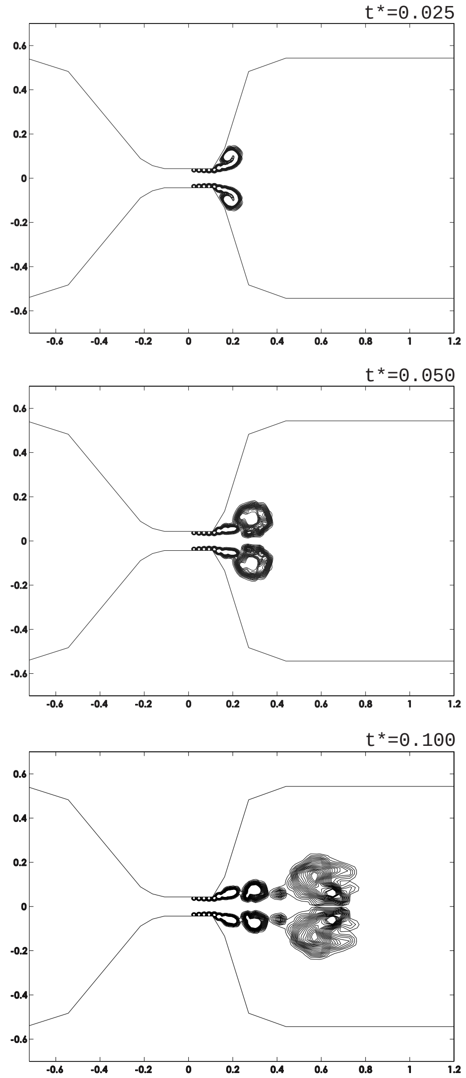

Figure 5:*Evolution of the nondimensional vorticity for the Reynolds number of Re=1000.*

3.2.2. Diffusion step of the core radius

The diffusion step should be evaluated using the diffusion equation

$$\frac{\partial \omega}{\partial t} = \nu \nabla^2 \omega. \tag{22}$$

According to the explicit solution, this step can be represented by spreading the core radius of each vortex as

$$\frac{d\sigma}{dt} = \frac{(2.242)^2 \nu}{2\sigma}. \tag{23}$$

Though this method does not always converge in Eq. (19), it is simple and precise if the time-step of the simulation is selected at a sufficiently small value [9].

4. Simulation results

This section presents some simulation results of the glottal flow and the resulting pressure distribution using the proposed method. Here the Reynolds number is defined as

$$Re = \frac{UL}{\nu}, \tag{24}$$

where U and L respectively represent the characteristic velocity and length of the glottal flow as shown in Fig. 1. This dimensionless number determines the characteristics of the glottal flow. In this study, the Reynolds number is set at *Re*=1000. In the next subsection, we show the evolution pattern of the vorticity and pressure distribution of the glottal flow.

4.1. Evolution of the vorticity field

In the infinite strip region, the potential function which describes the glottal flow with boundary layer separation is represented using the irrotational element in Eq. (8), the rotatonal elements in Eq. (9), and the core function in Eq. (17) as

$$f = U\zeta + \sum_{i=1}^{n} f_{vi} f_{\sigma}(\zeta - \zeta_i). \tag{25}$$

The first term means the expiratory flow from the lungs and second one the vorticity resulting from the boundary layer separation. n is the number of point vortices and it is increased at each time step of the simulation.

Simulation results on the evolution of the nondimensional vorticity field are shown in Fig. 5. We can see the process of the boundary layer separation and the growth of the vorticity field.

4.2. Pressure distribution of the glottal flow

It is important to estimate the pressure distribution around the glottal region to evaluate the interaction between the flow and vocal folds tissue. In this study, the pressure p is calculated using the unsteady Bernoulli equation as

$$p + \frac{1}{2}\rho U_0{}^2 + \rho \frac{\partial \phi}{\partial t} = F(t), \tag{26}$$

where ρ is the density of the air, $U_0 = |\overline{w}|$ is the magnitude of the complex velocity w in Eq. (13), ϕ is the velocity potential, and $F(t)$ is an arbitrary function of time t. ϕ is easily derived as

$$\phi = \mathrm{Re}(f) = \mathrm{Re}(\phi + i\varphi) \tag{27}$$

from the definition of the complex potential function in Eq. (3).

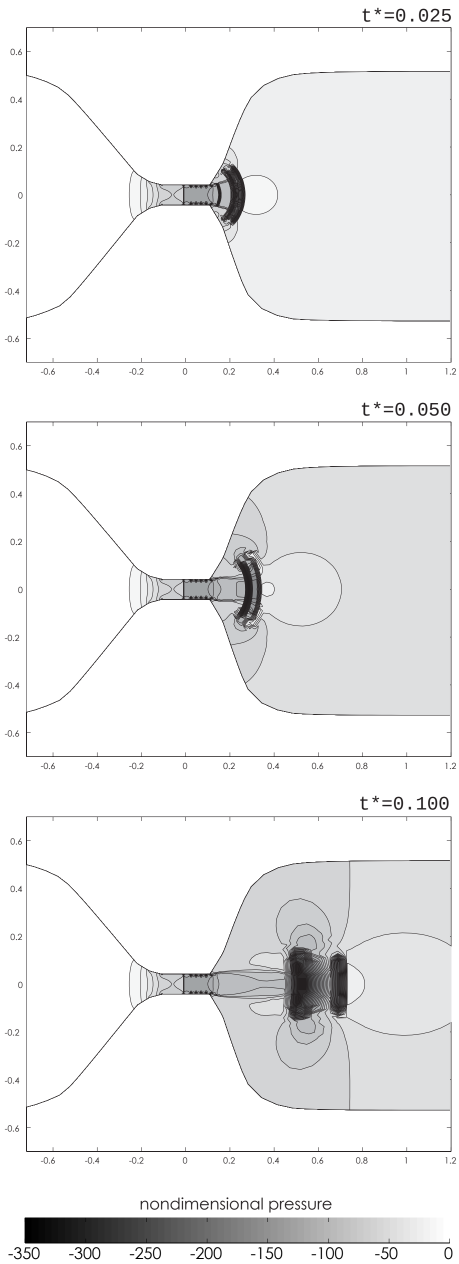

Figure 6: *Nondimensional pressure field calculated by the unsteady Bernoulli equation. Each time step corresponds to that shown in Fig. 5.*

Simulation results of the nondimensional pressure field are shown in Fig. 6. It can be seen from the figure that the vortex-related momentum change causes the pressure drop across the glottis. Also, the pressure difference between the sub- and supra-glottal regions increases from moment to moment.

5. Conclusions

This paper presented a method to represent the flow through the glottis using irrotational and rotational flow elements. In this method, the actual physical domain was connected to a canonical domain configured as the infinite strip by the mapping function obtained from the Schwarz-Christoffel formula. Potential functions were derived in the canonical domain and they were interpreted in the physical domain to represent the irrotational and rotational flow elements. The boundary layer separation was represented using the vortex method as the process in which a series of rotational flow elements were released at the separation point. By taking the dynamics of these elements into account, convection and diffusion of the vorticity field were represented. Also, the pressure distribution was evaluated using the unsteady Bernoulli equation. Experiments were conducted to simulate the evolution of the vorticity and pressure distribution. Further study will be performed to determine the separation point of the boundary layer and consider the dynamic motion of the vocal folds driven by the calculated pressure distribution.

6. References

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